Lecture Notes in

Electric Circuits (I)

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1.1 INTRODUCTION

Electric circuit theory and electromagnetic theory are the two fundamental theories upon which all branches of electrical engineering are built. Many branches of electrical engineering, such as power, electric machines, control, electronics, communications, and instrumentation, are based on electric circuit theory. Therefore, the basic electric circuit theory course is the most important course for an electrical engineering student, and always an excellent starting point for a beginning student in electrical engineering education. Circuit theory is also valuable to students specializing in other branches of the physical sciences because circuits are a good model for the study of energy systems in general, and because of the applied mathematics, physics, and topology involved.

In electrical engineering, we are often interested in communicating or transferring energy from one point to another. To do this requires an interconnection of electrical devices. Such interconnection is referred to as an *electric circuit*, and each component of the circuit is known as an *element.*



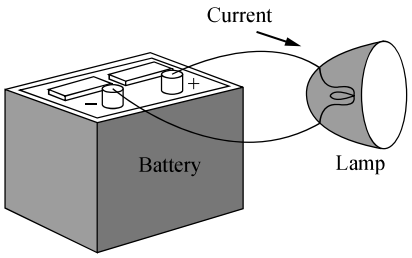


Figure 1.1 A simple electric circuit.

A simple electric circuit is shown in Fig. 1.1. It consists of three basic components: a battery, a lamp, and connecting wires. Such a simple circuit can exist by itself; it has several applications, such as a torch light, a search light, and so forth.

A complicated real circuit is displayed in Fig. 1.2, representing the schematic diagram for a radio receiver. Although it seems complicated, this circuit can be analyzed using the techniques we cover in this book. Our goal in this text is to learn various analytical techniques and computer software applications for describing the behavior of a circuit like this.

Electric circuits are used in numerous electrical systems to accomplish different tasks. Our objective in this book is not the study of various uses and applications of circuits. Rather our major concern is the analysis of the circuits. By the analysis of a circuit, we mean a study of the behavior of the circuit: How does it respond to a given input? How do the interconnected elements and devices in the circuit interact?

We commence our study by defining some basic concepts. These concepts include charge, current, voltage, circuit elements, power, and energy. Before defining these concepts, we must first establish a system of units that we will use throughout the text.

1.2 SYSTEMS OF UNITS

As electrical engineers, we deal with measurable quantities. Our measurement, however, must be communicated in a standard language that virtually all professionals can understand, irrespective of the country where the measurement is conducted. Such an international measurement language is the International System of Units (SI), adopted by the General Conference on Weights and Measures in 1960. In this system,

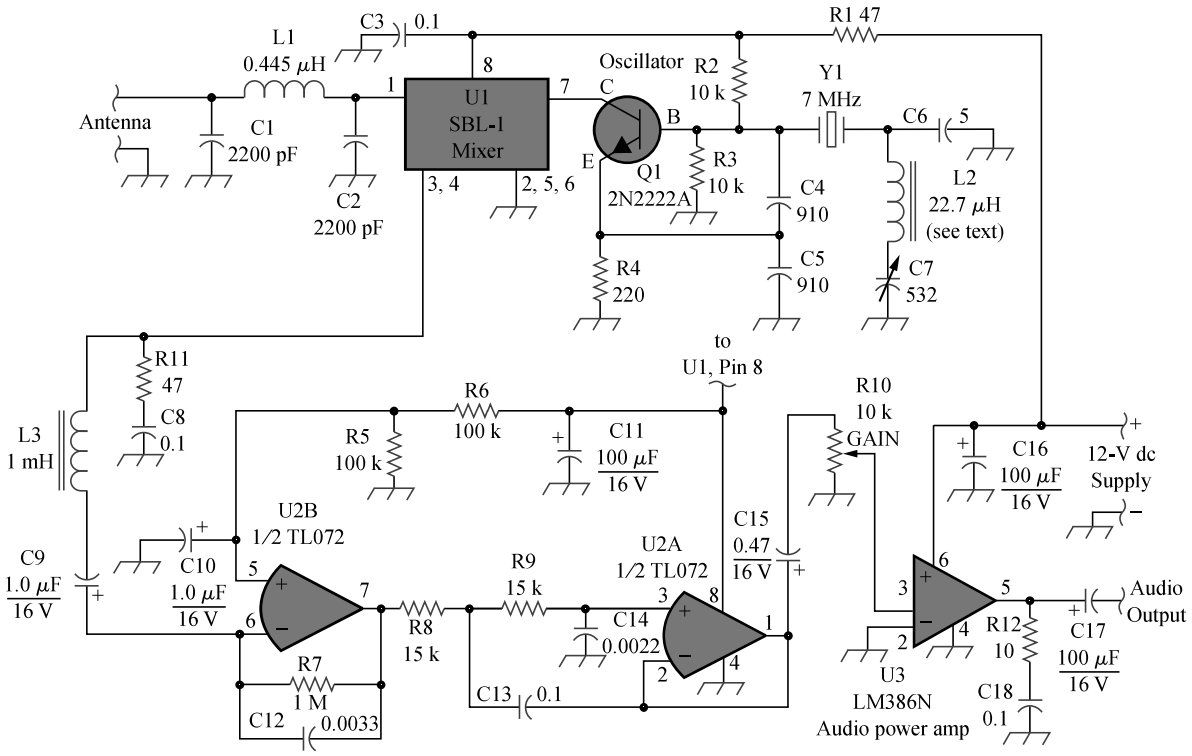


Figure 1.2 Electric circuit of a radio receiver.
(Reproduced with permission from QST, August 1995, p. 23.)

there are six principal units from which the units of all other physical quantities can be derived. Table 1.1 shows the six units, their symbols, and the physical quantities they represent. The SI units are used throughout this text.

One great advantage of the SI unit is that it uses prefixes based on the power of 10 to relate larger and smaller units to the basic unit. Table 1.2 shows the SI prefixes and their symbols. For example, the following are expressions of the same distance in meters (m):

600,000,000 mm 600,000 m 600 km

TABLE 1.1 The six basic SI units.

Quantity	Basic unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd

TABLE 1.2 The SI prefixes.

Multiplier	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

1.3 CHARGE AND CURRENT

The concept of electric charge is the underlying principle for explaining all electrical phenomena. Also, the most basic quantity in an electric circuit is the *electric charge*. We all experience the effect of electric charge when we try to remove our wool sweater and have it stick to our body or walk across a carpet and receive a shock.

Charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C).

We know from elementary physics that all matter is made of fundamental building blocks known as atoms and that each atom consists of electrons, protons, and neutrons. We also know that the charge e on an electron is negative and equal in magnitude to 1.602×10^{-19} C, while a proton carries a positive charge of the same magnitude as the electron. The presence of equal numbers of protons and electrons leaves an atom neutrally charged.

The following points should be noted about electric charge:

1. The coulomb is a large unit for charges. In 1 C of charge, there are $1/(1.602 \times 10^{-19}) = 6.24 \times 10^{18}$ electrons. Thus realistic or laboratory values of charges are on the order of pC, nC, or μC.[1]
2. According to experimental observations, the only charges that occur in nature are integral multiples of the electronic charge $e = -1.602 \times 10^{-19}$ C.
3. The *law of conservation of charge* states that charge can neither be created nor destroyed, only transferred. Thus the algebraic sum of the electric charges in a system does not change.

We now consider the flow of electric charges. A unique feature of electric charge or electricity is the fact that it is mobile; that is, it can be transferred from one place to another, where it can be converted to another form of energy.

When a conducting wire (consisting of several atoms) is connected to a battery (a source of electromotive force), the charges are compelled to move; positive charges move in one direction while negative charges move in the opposite direction. This motion of charges creates electric current. It is conventional to take the current flow as the movement of positive charges, that is, opposite to the flow of negative charges, as Fig. 1.3 illustrates. This convention was introduced by Benjamin Franklin (1706–1790), the American scientist and inventor. Although we now know that current in metallic conductors is due to negatively charged electrons, we will follow the universally accepted convention that current is the net flow of positive charges. Thus,

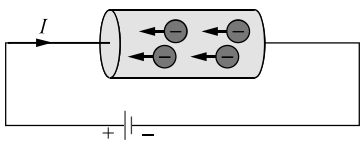


Figure 1.3 Electric current due to flow of electronic charge in a conductor.

A convention is a standard way of describing something so that others in the profession can understand what we mean. We will be using IEEE conventions throughout this book.

[1] However, a large power supply capacitor can store up to 0.5 C of charge.

Electric current is the time rate of change of charge, measured in amperes (A).

Mathematically, the relationship between current i, charge q, and time t is

$$i = \frac{dq}{dt} \tag{1.1}$$

where current is measured in amperes (A), and

$$1 \text{ ampere} = 1 \text{ coulomb/second}$$

The charge transferred between time t_0 and t is obtained by integrating both sides of Eq. (1.1). We obtain

$$q = \int_{t_0}^{t} i\, dt \tag{1.2}$$

The way we define current as i in Eq. (1.1) suggests that current need not be a constant-valued function. As many of the examples and problems in this chapter and subsequent chapters suggest, there can be several types of current; that is, charge can vary with time in several ways that may be represented by different kinds of mathematical functions.

If the current does not change with time, but remains constant, we call it a *direct current* (dc).

A direct current (dc) is a current that remains constant with time.

By convention the symbol I is used to represent such a constant current.

A time-varying current is represented by the symbol i. A common form of time-varying current is the sinusoidal current or *alternating current* (ac).

An alternating current (ac) is a current that varies sinusoidally with time.

Such current is used in your household, to run the air conditioner, refrigerator, washing machine, and other electric appliances. Figure 1.4 shows direct current and alternating current; these are the two most common types of current. We will consider other types later in the book.

Once we define current as the movement of charge, we expect current to have an associated direction of flow. As mentioned earlier, the direction of current flow is conventionally taken as the direction of positive charge movement. Based on this convention, a current of 5 A may be represented positively or negatively as shown in Fig. 1.5. In other words, a negative current of -5 A flowing in one direction as shown in Fig. 1.5(b) is the same as a current of $+5$ A flowing in the opposite direction.

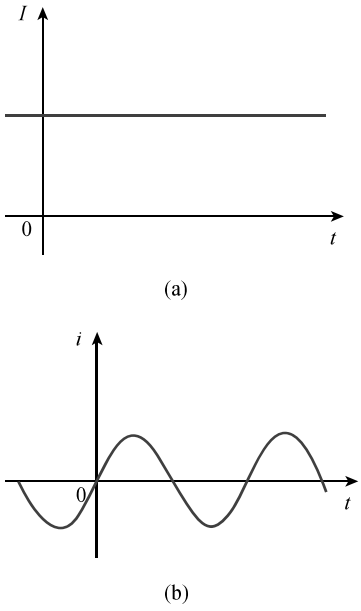


Figure 1.4 Two common types of current: (a) direct current (dc), (b) alternating current (ac).

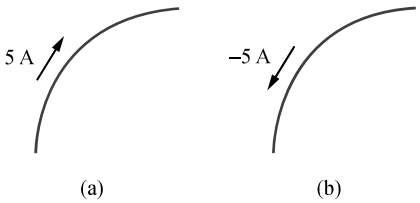


Figure 1.5 Conventional current flow: (a) positive current flow, (b) negative current flow.

EXAMPLE 1.1

How much charge is represented by 4,600 electrons?

Solution:

Each electron has -1.602×10^{-19} C. Hence 4,600 electrons will have

$$-1.602 \times 10^{-19} \text{ C/electron} \times 4{,}600 \text{ electrons} = -7.369 \times 10^{-16} \text{ C}$$

PRACTICE PROBLEM 1.1

Calculate the amount of charge represented by two million protons.

Answer: $+3.204 \times 10^{-13}$ C.

EXAMPLE 1.2

The total charge entering a terminal is given by $q = 5t \sin 4\pi t$ mC. Calculate the current at $t = 0.5$ s.

Solution:

$$i = \frac{dq}{dt} = \frac{d}{dt}(5t \sin 4\pi t) \text{ mC/s} = (5 \sin 4\pi t + 20\pi t \cos 4\pi t) \text{ mA}$$

At $t = 0.5$,

$$i = 5 \sin 2\pi + 10\pi \cos 2\pi = 0 + 10\pi = 31.42 \text{ mA}$$

PRACTICE PROBLEM 1.2

If in Example 1.2, $q = (10 - 10e^{-2t})$ mC, find the current at $t = 0.5$ s.

Answer: 7.36 mA.

EXAMPLE 1.3

Determine the total charge entering a terminal between $t = 1$ s and $t = 2$ s if the current passing the terminal is $i = (3t^2 - t)$ A.

Solution:

$$q = \int_{t=1}^{2} i\,dt = \int_{1}^{2} (3t^2 - t)\,dt$$

$$= \left(t^3 - \frac{t^2}{2}\right)\Bigg|_1^2 = (8 - 2) - \left(1 - \frac{1}{2}\right) = 5.5 \text{ C}$$

PRACTICE PROBLEM 1.3

The current flowing through an element is

$$i = \begin{cases} 2 \text{ A}, & 0 < t < 1 \\ 2t^2 \text{ A}, & t > 1 \end{cases}$$

Calculate the charge entering the element from $t = 0$ to $t = 2$ s.

Answer: 6.667 C.

1.4 VOLTAGE

As explained briefly in the previous section, to move the electron in a conductor in a particular direction requires some work or energy transfer. This work is performed by an external electromotive force (emf), typically represented by the battery in Fig. 1.3. This emf is also known as *voltage* or *potential difference*. The voltage v_{ab} between two points a and b in an electric circuit is the energy (or work) needed to move a unit charge from a to b; mathematically,

$$v_{ab} = \frac{dw}{dq} \tag{1.3}$$

where w is energy in joules (J) and q is charge in coulombs (C). The voltage v_{ab} or simply v is measured in volts (V), named in honor of the Italian physicist Alessandro Antonio Volta (1745–1827), who invented the first voltaic battery. From Eq. (1.3), it is evident that

$$1 \text{ volt} = 1 \text{ joule/coulomb} = 1 \text{ newton meter/coulomb}$$

Thus,

Voltage (or potential difference) is the energy required to move a unit charge through an element, measured in volts (V).

Figure 1.6 shows the voltage across an element (represented by a rectangular block) connected to points a and b. The plus (+) and minus (−) signs are used to define reference direction or voltage polarity. The v_{ab} can be interpreted in two ways: (1) point a is at a potential of v_{ab} volts higher than point b, or (2) the potential at point a with respect to point b is v_{ab}. It follows logically that in general

$$v_{ab} = -v_{ba} \tag{1.4}$$

For example, in Fig. 1.7, we have two representations of the same voltage. In Fig. 1.7(a), point a is +9 V above point b; in Fig. 1.7(b), point b is −9 V above point a. We may say that in Fig. 1.7(a), there is a 9-V *voltage drop* from a to b or equivalently a 9-V *voltage rise* from b to a. In other words, a voltage drop from a to b is equivalent to a voltage rise from b to a.

Current and voltage are the two basic variables in electric circuits. The common term *signal* is used for an electric quantity such as a current or a voltage (or even electromagnetic wave) when it is used for conveying information. Engineers prefer to call such variables signals rather than mathematical functions of time because of their importance in communications and other disciplines. Like electric current, a constant voltage is called a *dc voltage* and is represented by V, whereas a sinusoidally time-varying voltage is called an *ac voltage* and is represented by v. A dc voltage is commonly produced by a battery; ac voltage is produced by an electric generator.

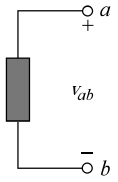


Figure 1.6 Polarity of voltage v_{ab}.

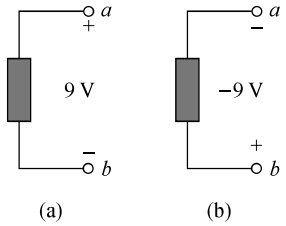


Figure 1.7 Two equivalent representations of the same voltage v_{ab}: (a) point a is 9 V above point b, (b) point b is −9 V above point a.

Keep in mind that electric current is always *through* an element and that electric voltage is always *across* the element or between two points.

1.5 POWER AND ENERGY

Although current and voltage are the two basic variables in an electric circuit, they are not sufficient by themselves. For practical purposes, we need to know how much *power* an electric device can handle. We all know from experience that a 100-watt bulb gives more light than a 60-watt bulb. We also know that when we pay our bills to the electric utility companies, we are paying for the electric *energy* consumed over a certain period of time. Thus power and energy calculations are important in circuit analysis.

To relate power and energy to voltage and current, we recall from physics that:

> Power is the time rate of expending or absorbing energy, measured in watts (W).

We write this relationship as

$$p = \frac{dw}{dt} \tag{1.5}$$

where p is power in watts (W), w is energy in joules (J), and t is time in seconds (s). From Eqs. (1.1), (1.3), and (1.5), it follows that

$$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = vi \tag{1.6}$$

or

$$p = vi \tag{1.7}$$

The power p in Eq. (1.7) is a time-varying quantity and is called the *instantaneous power*. Thus, the power absorbed or supplied by an element is the product of the voltage across the element and the current through it. If the power has a + sign, power is being delivered to or absorbed by the element. If, on the other hand, the power has a − sign, power is being supplied by the element. But how do we know when the power has a negative or a positive sign?

Current direction and voltage polarity play a major role in determining the sign of power. It is therefore important that we pay attention to the relationship between current i and voltage v in Fig. 1.8(a). The voltage polarity and current direction must conform with those shown in Fig. 1.8(a) in order for the power to have a positive sign. This is known as the *passive sign convention.* By the passive sign convention, current enters through the positive polarity of the voltage. In this case, $p = +vi$ or $vi > 0$ implies that the element is absorbing power. However, if $p = -vi$ or $vi < 0$, as in Fig. 1.8(b), the element is releasing or supplying power.

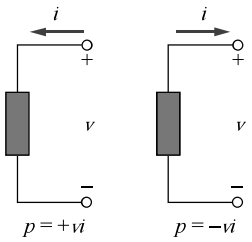


Figure 1.8 Reference polarities for power using the passive sign convention: (a) absorbing power, (b) supplying power.

> Passive sign convention is satisfied when the current enters through the positive terminal of an element and $p = +vi$. If the current enters through the negative terminal, $p = -vi$.

Unless otherwise stated, we will follow the passive sign convention throughout this text. For example, the element in both circuits of Fig. 1.9 has an absorbing power of +12 W because a positive current enters the positive terminal in both cases. In Fig. 1.10, however, the element is supplying power of −12 W because a positive current enters the negative terminal. Of course, an absorbing power of +12 W is equivalent to a supplying power of −12 W. In general,

When the voltage and current directions conform to Fig. 1.8(b), we have the *active sign convention* and $p = +vi$.

$$\text{Power absorbed} = -\text{Power supplied}$$

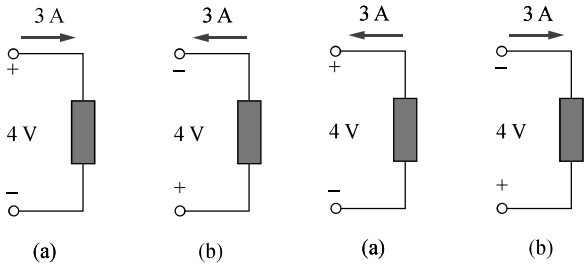


Figure 1.9 Two cases of an element with an absorbing power of 12 W:
(a) $p = 4 \times 3 = 12$ W,
(b) $p = 4 \times 3 = 12$ W.

Figure 1.10 Two cases of an element with a supplying power of 12 W:
(a) $p = 4 \times (-3) = -12$ W,
(b) $p = 4 \times (-3) = -12$ W.

In fact, the *law of conservation of energy* must be obeyed in any electric circuit. For this reason, the algebraic sum of power in a circuit, at any instant of time, must be zero:

$$\boxed{\sum p = 0} \tag{1.8}$$

This again confirms the fact that the total power supplied to the circuit must balance the total power absorbed.

From Eq. (1.6), the energy absorbed or supplied by an element from time t_0 to time t is

$$w = \int_{t_0}^{t} p\,dt = \int_{t_0}^{t} vi\,dt \tag{1.9}$$

Energy is the capacity to do work, measured in joules (J).

The electric power utility companies measure energy in watt-hours (Wh), where

$$1 \text{ Wh} = 3{,}600 \text{ J}$$

EXAMPLE 1.4

An energy source forces a constant current of 2 A for 10 s to flow through a lightbulb. If 2.3 kJ is given off in the form of light and heat energy, calculate the voltage drop across the bulb.

Solution:

The total charge is

$$\Delta q = i\,\Delta t = 2 \times 10 = 20 \text{ C}$$

The voltage drop is

$$v = \frac{\Delta w}{\Delta q} = \frac{2.3 \times 10^3}{20} = 115 \text{ V}$$

PRACTICE PROBLEM 1.4

To move charge q from point a to point b requires -30 J. Find the voltage drop v_{ab} if: (a) $q = 2$ C, (b) $q = -6$ C .

Answer: (a) -15 V, (b) 5 V.

EXAMPLE 1.5

Find the power delivered to an element at $t = 3$ ms if the current entering its positive terminal is

$$i = 5\cos 60\pi t \text{ A}$$

and the voltage is: (a) $v = 3i$, (b) $v = 3\,di/dt$.

Solution:

(a) The voltage is $v = 3i = 15\cos 60\pi t$; hence, the power is

$$p = vi = 75\cos^2 60\pi t \text{ W}$$

At $t = 3$ ms,

$$p = 75\cos^2(60\pi \times 3 \times 10^{-3}) = 75\cos^2 0.18\pi = 53.48 \text{ W}$$

(b) We find the voltage and the power as

$$v = 3\frac{di}{dt} = 3(-60\pi)5\sin 60\pi t = -900\pi \sin 60\pi t \text{ V}$$

$$p = vi = -4500\pi \sin 60\pi t \cos 60\pi t \text{ W}$$

At $t = 3$ ms,

$$p = -4500\pi \sin 0.18\pi \cos 0.18\pi \text{ W}$$
$$= -14137.167 \sin 32.4^\circ \cos 32.4^\circ = -6.396 \text{ kW}$$

PRACTICE PROBLEM 1.5

Find the power delivered to the element in Example 1.5 at $t = 5$ ms if the current remains the same but the voltage is: (a) $v = 2i$ V, (b) $v = \left(10 + 5\int_0^t i\,dt\right)$ V.

Answer: (a) 17.27 W, (b) 29.7 W.

EXAMPLE 1.6

How much energy does a 100-W electric bulb consume in two hours?

Solution:

$$w = pt = 100\ (\text{W}) \times 2\ (\text{h}) \times 60\ (\text{min/h}) \times 60\ (\text{s/min})$$
$$= 720{,}000\ \text{J} = 720\ \text{kJ}$$

This is the same as

$$w = pt = 100\ \text{W} \times 2\ \text{h} = 200\ \text{Wh}$$



PRACTICE PROBLEM 1.6

A stove element draws 15 A when connected to a 120-V line. How long does it take to consume 30 kJ?

Answer: 16.67 s.

1.6 CIRCUIT ELEMENTS

As we discussed in Section 1.1, an element is the basic building block of a circuit. An electric circuit is simply an interconnection of the elements. Circuit analysis is the process of determining voltages across (or the currents through) the elements of the circuit.

There are two types of elements found in electric circuits: *passive* elements and *active* elements. An active element is capable of generating energy while a passive element is not. Examples of passive elements are resistors, capacitors, and inductors. Typical active elements include generators, batteries, and operational amplifiers. Our aim in this section is to gain familiarity with some important active elements.

The most important active elements are voltage or current sources that generally deliver power to the circuit connected to them. There are two kinds of sources: independent and dependent sources.

An ideal independent source is an active element that provides a specified voltage or current that is completely independent of other circuit variables.

In other words, an ideal independent voltage source delivers to the circuit whatever current is necessary to maintain its terminal voltage. Physical sources such as batteries and generators may be regarded as approximations to ideal voltage sources. Figure 1.11 shows the symbols for independent voltage sources. Notice that both symbols in Fig. 1.11(a) and (b) can be used to represent a dc voltage source, but only the symbol in Fig. 1.11(a) can be used for a time-varying voltage source. Similarly, an ideal independent current source is an active element that provides a specified current completely independent of the voltage across the source. That is, the current source delivers to the circuit whatever voltage is necessary to

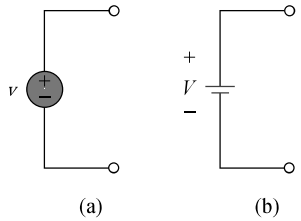


Figure 1.11 Symbols for independent voltage sources: (a) used for constant or time-varying voltage, (b) used for constant voltage (dc).

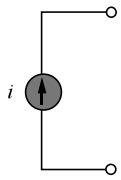


Figure 1.12 Symbol for independent current source.

maintain the designated current. The symbol for an independent current source is displayed in Fig. 1.12, where the arrow indicates the direction of current i.

An ideal dependent (or controlled) source is an active element in which the source quantity is controlled by another voltage or current.

Dependent sources are usually designated by diamond-shaped symbols, as shown in Fig. 1.13. Since the control of the dependent source is achieved by a voltage or current of some other element in the circuit, and the source can be voltage or current, it follows that there are four possible types of dependent sources, namely:

1. A voltage-controlled voltage source (VCVS).
2. A current-controlled voltage source (CCVS).
3. A voltage-controlled current source (VCCS).
4. A current-controlled current source (CCCS).

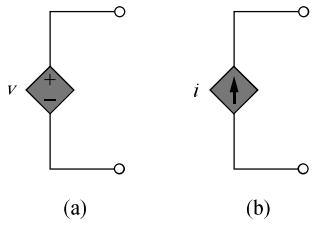


Figure 1.13 Symbols for: (a) dependent voltage source, (b) dependent current source.

Dependent sources are useful in modeling elements such as transistors, operational amplifiers and integrated circuits. An example of a current-controlled voltage source is shown on the right-hand side of Fig. 1.14, where the voltage $10i$ of the voltage source depends on the current i through element C. Students might be surprised that the value of the dependent voltage source is $10i$ V (and not $10i$ A) because it is a voltage source. The key idea to keep in mind is that a voltage source comes with polarities $(+\ -)$ in its symbol, while a current source comes with an arrow, irrespective of what it depends on.

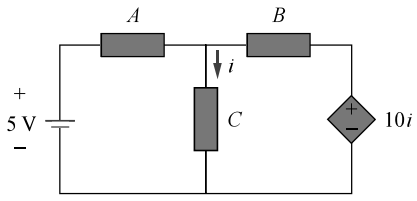


Figure 1.14 The source on the right-hand side is a current-controlled voltage source.

It should be noted that an ideal voltage source (dependent or independent) will produce any current required to ensure that the terminal voltage is as stated, whereas an ideal current source will produce the necessary voltage to ensure the stated current flow. Thus an ideal source could in theory supply an infinite amount of energy. It should also be noted that not only do sources supply power to a circuit, they can absorb power from a circuit too. For a voltage source, we know the voltage but not the current supplied or drawn by it. By the same token, we know the current supplied by a current source but not the voltage across it.

EXAMPLE 1.7

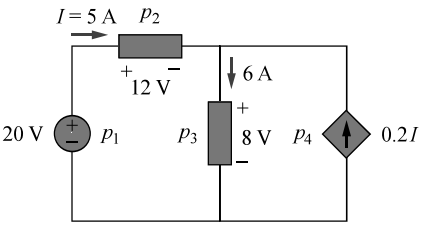


Figure 1.15 For Example 1.7.

Calculate the power supplied or absorbed by each element in Fig. 1.15.

Solution:

We apply the sign convention for power shown in Figs. 1.8 and 1.9. For p_1, the 5-A current is out of the positive terminal (or into the negative terminal); hence,

$$p_1 = 20(-5) = -100 \text{ W} \qquad \text{Supplied power}$$

For p_2 and p_3, the current flows into the positive terminal of the element in each case.

$$p_2 = 12(5) = 60 \text{ W} \qquad \text{Absorbed power}$$
$$p_3 = 8(6) = 48 \text{ W} \qquad \text{Absorbed power}$$

For p_4, we should note that the voltage is 8 V (positive at the top), the same as the voltage for p_3, since both the passive element and the dependent source are connected to the same terminals. (Remember that voltage is always measured across an element in a circuit.) Since the current flows out of the positive terminal,

$$p_4 = 8(-0.2I) = 8(-0.2 \times 5) = -8 \text{ W} \qquad \text{Supplied power}$$

We should observe that the 20-V independent voltage source and $0.2I$ dependent current source are supplying power to the rest of the network, while the two passive elements are absorbing power. Also,

$$p_1 + p_2 + p_3 + p_4 = -100 + 60 + 48 - 8 = 0$$

In agreement with Eq. (1.8), the total power supplied equals the total power absorbed.

PRACTICE PROBLEM 1.7

Compute the power absorbed or supplied by each component of the circuit in Fig. 1.16.

Answer: $p_1 = -40$ W, $p_2 = 16$ W, $p_3 = 9$ W, $p_4 = 15$ W.

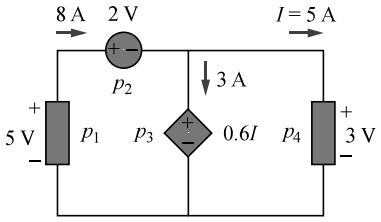


Figure 1.16 For Practice Prob. 1.7.

†1.7 APPLICATIONS[2]

In this section, we will consider two practical applications of the concepts developed in this chapter. The first one deals with the TV picture tube and the other with how electric utilities determine your electric bill.

1.7.1 TV Picture Tube

One important application of the motion of electrons is found in both the transmission and reception of TV signals. At the transmission end, a TV camera reduces a scene from an optical image to an electrical signal. Scanning is accomplished with a thin beam of electrons in an iconoscope camera tube.

At the receiving end, the image is reconstructed by using a cathode-ray tube (CRT) located in the TV receiver.[3] The CRT is depicted in

[2]The dagger sign preceding a section heading indicates a section that may be skipped, explained briefly, or assigned as homework.

[3]Modern TV tubes use a different technology.

Fig. 1.17. Unlike the iconoscope tube, which produces an electron beam of constant intensity, the CRT beam varies in intensity according to the incoming signal. The electron gun, maintained at a high potential, fires the electron beam. The beam passes through two sets of plates for vertical and horizontal deflections so that the spot on the screen where the beam strikes can move right and left and up and down. When the electron beam strikes the fluorescent screen, it gives off light at that spot. Thus the beam can be made to "paint" a picture on the TV screen.

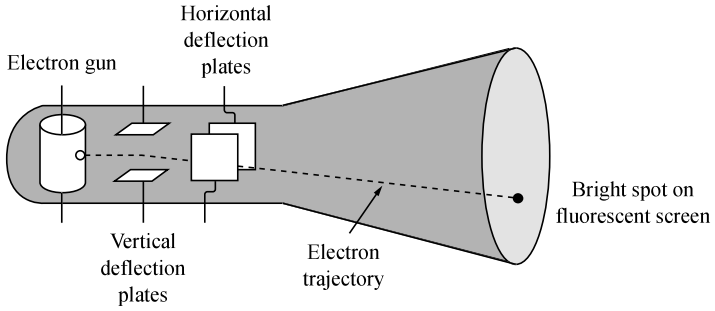


Figure 1.17 Cathode-ray tube.
(Source: D. E. Tilley, Contemporary College Physics [Menlo Park, CA: Benjamin/Cummings, 1979], p. 319.)

EXAMPLE 1.8

The electron beam in a TV picture tube carries 10^{15} electrons per second. As a design engineer, determine the voltage V_o needed to accelerate the electron beam to achieve 4 W.

Solution:

The charge on an electron is

$$e = -1.6 \times 10^{-19} \text{ C}$$

If the number of electrons is n, then $q = ne$ and

$$i = \frac{dq}{dt} = e\frac{dn}{dt} = (-1.6 \times 10^{-19})(10^{15}) = -1.6 \times 10^{-4} \text{ A}$$

The negative sign indicates that the electron flows in a direction opposite to electron flow as shown in Fig. 1.18, which is a simplified diagram of the CRT for the case when the vertical deflection plates carry no charge. The beam power is

$$p = V_o i \qquad \text{or} \qquad V_o = \frac{p}{i} = \frac{4}{1.6 \times 10^{-4}} = 25{,}000 \text{ V}$$

Thus the required voltage is 25 kV.

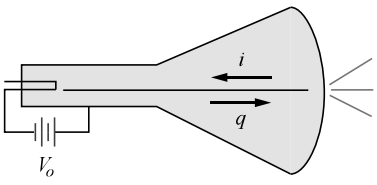


Figure 1.18 A simplified diagram of the cathode-ray tube; for Example 1.8.

PRACTICE PROBLEM 1.8

If an electron beam in a TV picture tube carries 10^{13} electrons/second and is passing through plates maintained at a potential difference of 30 kV, calculate the power in the beam.

Answer: 48 mW.

1.7.2 Electricity Bills

The second application deals with how an electric utility company charges their customers. The cost of electricity depends upon the amount of energy consumed in kilowatt-hours (kWh). (Other factors that affect the cost include demand and power factors; we will ignore these for now.) However, even if a consumer uses no energy at all, there is a minimum service charge the customer must pay because it costs money to stay connected to the power line. As energy consumption increases, the cost per kWh drops. It is interesting to note the average monthly consumption of household appliances for a family of five, shown in Table 1.3.

TABLE 1.3 Typical average monthly consumption of household appliances.

Appliance	kWh consumed	Appliance	kWh consumed
Water heater	500	Washing machine	120
Freezer	100	Stove	100
Lighting	100	Dryer	80
Dishwasher	35	Microwave oven	25
Electric iron	15	Personal computer	12
TV	10	Radio	8
Toaster	4	Clock	2

EXAMPLE 1.9

A homeowner consumes 3,300 kWh in January. Determine the electricity bill for the month using the following residential rate schedule:

Base monthly charge of $12.00.

First 100 kWh per month at 16 cents/kWh.

Next 200 kWh per month at 10 cents/kWh.

Over 200 kWh per month at 6 cents/kWh.

Solution:

We calculate the electricity bill as follows.

$$\begin{aligned} \text{Base monthly charge} &= \$12.00 \\ \text{First 100 kWh @ \$0.16/kWh} &= \$16.00 \\ \text{Next 200 kWh @ \$0.10/kWh} &= \$20.00 \\ \text{Remaining 100 kWh @ \$0.06/kWh} &= \$6.00 \\ \text{Total Charge} &= \$54.00 \end{aligned}$$

$$\text{Average cost} = \frac{\$54}{100 + 200 + 100} = 13.5 \text{ cents/kWh}$$

PRACTICE PROBLEM 1.9

Referring to the residential rate schedule in Example 1.9, calculate the average cost per kWh if only 400 kWh are consumed in July when the family is on vacation most of the time.

Answer: 13.5 cents/kWh.

†1.8 PROBLEM SOLVING

Although the problems to be solved during one's career will vary in complexity and magnitude, the basic principles to be followed remain the same. The process outlined here is the one developed by the authors over many years of problem solving with students, for the solution of engineering problems in industry, and for problem solving in research.

We will list the steps simply and then elaborate on them.

1. Carefully **Define** the problem.
2. **Present** everything you know about the problem.
3. Establish a set of **Alternative** solutions and determine the one that promises the greatest likelihood of success.
4. **Attempt** a problem solution.
5. **Evaluate** the solution and check for accuracy.
6. Has the problem been solved **Satisfactorily**? If so, present the solution; if not, then return to step 3 and continue through the process again.

1. *Carefully **Define** the problem*. This may be the most important part of the process, because it becomes the foundation for all the rest of the steps. In general, the presentation of engineering problems is somewhat incomplete. You must do all you can to make sure you understand the problem as thoroughly as the presenter of the problem understands it. Time spent at this point clearly identifying the problem will save you considerable time and frustration later. As a student, you can clarify a problem statement in a textbook by asking your professor to help you understand it better. A problem presented to you in industry may require that you consult several individuals. At this step, it is important to develop questions that need to be addressed before continuing the solution process. If you have such questions, you need to consult with the appropriate individuals or resources to obtain the answers to those questions. With those answers, you can now refine the problem, and use that refinement as the problem statement for the rest of the solution process.

2. ***Present** everything you know about the problem*. You are now ready to write down everything you know about the problem and its possible solutions. This important step will save you time and frustration later.

3. *Establish a set of **Alternative** solutions and determine the one that promises the greatest likelihood of success.* Almost every problem will have a number of possible paths that can lead to a solution. It is highly desirable to identify as many of those paths as possible. At this point, you also need to determine what tools are available to you, such as Matlab and other software packages that can greatly reduce effort and increase accuracy. Again, we want to stress that time spent carefully defining the problem and investigating alternative approaches to its solution will pay big dividends later. Evaluating the alternatives and determining which promises the greatest likelihood of success may be difficult but will be well worth the effort. Document this process well since you will want to come back to it if the first approach does not work.

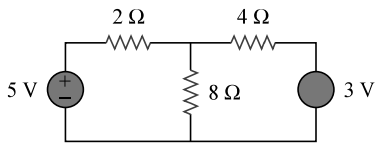
4. ***Attempt** a problem solution.* Now is the time to actually begin solving the problem. The process you follow must be well documented in order to present a detailed solution if successful, and to evaluate the process if you are not successful. This detailed evaluation may lead to corrections that can then lead to a successful solution. It can also lead to new alternatives to try. Many times, it is wise to fully set up a solution before putting numbers into equations. This will help in checking your results.

5. ***Evaluate** the solution and check for accuracy.* You now thoroughly evaluate what you have accomplished. Decide if you have an acceptable solution, one that you want to present to your team, boss, or professor.

6. *Has the problem been solved **Satisfactorily**? If so, present the solution; if not, then return to step 3 and continue through the process again.* Now you need to present your solution or try another alternative. At this point, presenting your solution may bring closure to the process. Often, however, presentation of a solution leads to further refinement of the problem definition, and the process continues. Following this process will eventually lead to a satisfactory conclusion.

Now let us look at this process for a student taking an electrical and computer engineering foundations course. (The basic process also applies to almost every engineering course.) Keep in mind that although the steps have been simplified to apply to academic types of problems, the process as stated always needs to be followed. We consider a simple example.

Assume that we have been given the following circuit. The instructor asks us to solve for the current flowing through the 8-ohm resistor.

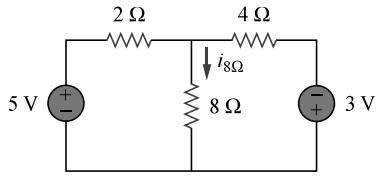


1. *Carefully **Define** the problem.* This is only a simple example, but we can already see that we do not know the polarity on the 3-V source. We have the following options. We can ask the professor what

the polarity should be. If we cannot ask, then we need to make a decision on what to do next. If we have time to work the problem both ways, we can solve for the current when the 3-V source is plus on top and then plus on the bottom. If we do not have the time to work it both ways, assume a polarity and then carefully document your decision. Let us assume that the professor tells us that the source is plus on the bottom.

2. ***Present*** *everything you know about the problem.* Presenting all that we know about the problem involves labeling the circuit clearly so that we define what we seek.

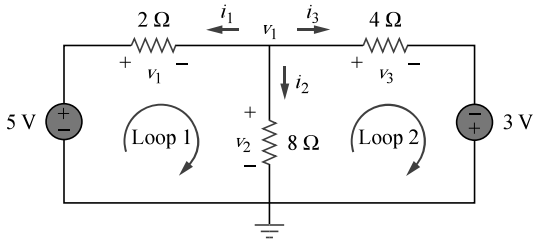
Given the following circuit, solve for $i_{8\Omega}$.



We now check with the professor, if reasonable, to see if the problem is properly defined.

3. *Establish a set of* ***Alternative*** *solutions and determine the one that promises the greatest likelihood of success.* There are essentially three techniques that can be used to solve this problem. Later in the text you will see that you can use circuit analysis (using Kirchoff's laws and Ohm's law), nodal analysis, and mesh analysis.

To solve for $i_{8\Omega}$ using circuit analysis will eventually lead to a solution, but it will likely take more work than either nodal or mesh analysis. To solve for $i_{8\Omega}$ using mesh analysis will require writing two simultaneous equations to find the two loop currents indicated in the following circuit. Using nodal analysis requires solving for only one unknown. This is the easiest approach.



Therefore, we will solve for $i_{8\Omega}$ using nodal analysis.

4. ***Attempt*** *a problem solution.* We first write down all of the equations we will need in order to find $i_{8\Omega}$.

$$i_{8\Omega} = i_2, \qquad i_2 = \frac{v_1}{8}, \qquad i_{8\Omega} = \frac{v_1}{8}$$

$$\frac{v_1 - 5}{2} + \frac{v_1 - 0}{8} + \frac{v_1 + 3}{4} = 0$$

Now we can solve for v_1.

$$8\left[\frac{v_1 - 5}{2} + \frac{v_1 - 0}{8} + \frac{v_1 + 3}{4}\right] = 0$$

leads to $(4v_1 - 20) + (v_1) + (2v_1 + 6) = 0$

$$7v_1 = +14, \qquad v_1 = +2 \text{ V}, \qquad i_{8\Omega} = \frac{v_1}{8} = \frac{2}{8} = \underline{\mathbf{0.25\ A}}$$

5. ***Evaluate*** *the solution and check for accuracy.* We can now use Kirchoff's voltage law to check the results.

$$i_1 = \frac{v_1 - 5}{2} = \frac{2 - 5}{2} = -\frac{3}{2} = -1.5 \text{ A}$$

$$i_2 = i_{8\Omega} = 0.25 \text{ A}$$

$$i_3 = \frac{v_1 + 3}{4} = \frac{2 + 3}{4} = \frac{5}{4} = 1.25 \text{ A}$$

$$i_1 + i_2 + i_3 = \underline{\mathbf{-1.5 + 0.25 + 1.25 = 0}} \qquad \text{(Checks.)}$$

Applying KVL to loop 1,

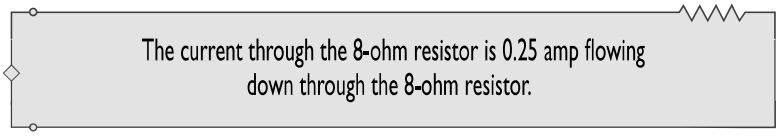
$$\begin{aligned} -5 + v_1 + v_2 &= -5 + (-i_1 \times 2) + (i_2 \times 8) \\ &= -5 + (-(-1.5)2) + (0.25 \times 8) \\ &= \underline{\mathbf{-5 + 3 + 2 = 0}} \qquad \text{(Checks.)} \end{aligned}$$

Applying KVL to loop 2,

$$\begin{aligned} -v_2 + v_3 - 3 &= -(i_2 \times 8) + (i_3 \times 4) - 3 \\ &= -(0.25 \times 8) + (1.25 \times 4) - 3 \\ &= \underline{\mathbf{-2 + 5 - 3 = 0}} \qquad \text{(Checks.)} \end{aligned}$$

So we now have a very high degree of confidence in the accuracy of our answer.

6. Has the problem been solved **Satisfactorily**? If so, present the solution; if not, then return to step 3 and continue through the process again. This problem has been solved satisfactorily.



1.9 SUMMARY

1. An electric circuit consists of electrical elements connected together.
2. The International System of Units (SI) is the international measurement language, which enables engineers to communicate their results. From the six principal units, the units of other physical quantities can be derived.
3. Current is the rate of charge flow.

$$i = \frac{dq}{dt}$$

4. Voltage is the energy required to move 1 C of charge through an element.

$$v = \frac{dw}{dq}$$

5. Power is the energy supplied or absorbed per unit time. It is also the product of voltage and current.

$$p = \frac{dw}{dt} = vi$$

6. According to the passive sign convention, power assumes a positive sign when the current enters the positive polarity of the voltage across an element.
7. An ideal voltage source produces a specific potential difference across its terminals regardless of what is connected to it. An ideal current source produces a specific current through its terminals regardless of what is connected to it.
8. Voltage and current sources can be dependent or independent. A dependent source is one whose value depends on some other circuit variable.
9. Two areas of application of the concepts covered in this chapter are the TV picture tube and electricity billing procedure.

REVIEW QUESTIONS

1.1 One millivolt is one millionth of a volt.
(a) True (b) False

1.2 The prefix *micro* stands for:
(a) 10^6 (b) 10^3 (c) 10^{-3} (d) 10^{-6}

1.3 The voltage 2,000,000 V can be expressed in powers of 10 as:
(a) 2 mV (b) 2 kV (c) 2 MV (d) 2 GV

1.4 A charge of 2 C flowing past a given point each second is a current of 2 A.
(a) True (b) False

1.5 A 4-A current charging a dielectric material will accumulate a charge of 24 C after 6 s.
(a) True (b) False

1.6 The unit of current is:
(a) Coulomb (b) Ampere
(c) Volt (d) Joule

1.7 Voltage is measured in:
(a) Watts (b) Amperes
(c) Volts (d) Joules per second

1.8 The voltage across a 1.1 kW toaster that produces a current of 10 A is:
(a) 11 kV (b) 1100 V (c) 110 V (d) 11 V

1.9 Which of these is not an electrical quantity?
(a) charge (b) time (c) voltage
(d) current (e) power

1.10 The dependent source in Fig. 1.19 is:
(a) voltage-controlled current source
(b) voltage-controlled voltage source
(c) current-controlled voltage source
(d) current-controlled current source

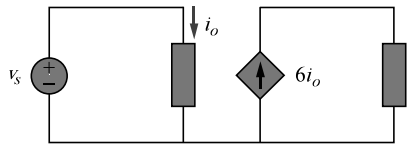


Figure 1.19 For Review Question 1.10.

Answers: 1.1b, 1.2d, 1.3c, 1.4a, 1.5a, 1.6b, 1.7c, 1.8c, 1.9b, 1.10d.

PROBLEMS

Section 1.3 Charge and Current

1.1 How many coulombs are represented by these amounts of electrons:

(a) 6.482×10^{17} (b) 1.24×10^{18}
(c) 2.46×10^{19} (d) 1.628×10^{20}

1.2 Find the current flowing through an element if the charge flow is given by:

(a) $q(t) = (t + 2)$ mC
(b) $q(t) = (5t^2 + 4t - 3)$ C
(c) $q(t) = 10e^{-4t}$ pC
(d) $q(t) = 20 \cos 50\pi t$ nC
(e) $q(t) = 5e^{-2t} \sin 100t$ μC

1.3 Find the charge $q(t)$ flowing through a device if the current is:

(a) $i(t) = 3$ A, $q(0) = 1$ C
(b) $i(t) = (2t + 5)$ mA, $q(0) = 0$
(c) $i(t) = 20 \cos(10t + \pi/6)$ μA, $q(0) = 2$ μC
(d) $i(t) = 10e^{-30t} \sin 40t$ A, $q(0) = 0$

1.4 The current flowing through a device is $i(t) = 5 \sin 6\pi t$ A. Calculate the total charge flow through the device from $t = 0$ to $t = 10$ ms.

1.5 Determine the total charge flowing into an element for $0 < t < 2$ s when the current entering its positive terminal is $i(t) = e^{-2t}$ mA.

1.6 The charge entering a certain element is shown in Fig. 1.20. Find the current at:

(a) $t = 1$ ms (b) $t = 6$ ms (c) $t = 10$ ms

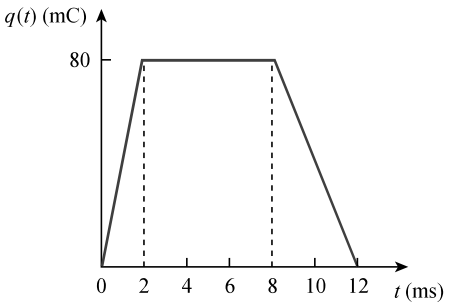


Figure 1.20 For Prob. 1.6.

1.7 The charge flowing in a wire is plotted in Fig. 1.21. Sketch the corresponding current.

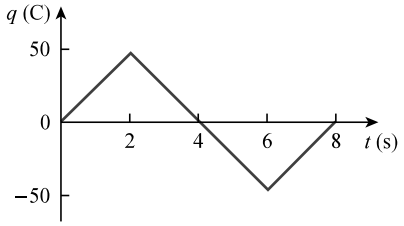


Figure 1.21 For Prob. 1.7.

1.8 The current flowing past a point in a device is shown in Fig. 1.22. Calculate the total charge through the point.

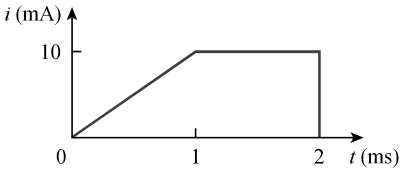


Figure 1.22 For Prob. 1.8.

1.9 The current through an element is shown in Fig. 1.23. Determine the total charge that passed through the element at:

(a) $t = 1$ s (b) $t = 3$ s (c) $t = 5$ s

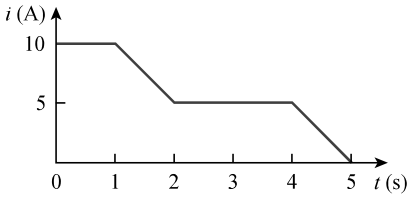


Figure 1.23 For Prob. 1.9.

Sections 1.4 and 1.5 Voltage, Power, and Energy

1.10 A certain electrical element draws the current $i(t) = 10 \cos 4t$ A at a voltage $v(t) = 120 \cos 4t$ V. Find the energy absorbed by the element in 2 s.

1.11 The voltage v across a device and the current i through it are

$$v(t) = 5 \cos 2t \text{ V}, \qquad i(t) = 10(1 - e^{-0.5t}) \text{ A}$$

Calculate:

(a) the total charge in the device at $t = 1$ s
(b) the power consumed by the device at $t = 1$ s.

1.12 The current entering the positive terminal of a device is $i(t) = 3e^{-2t}$ A and the voltage across the device is $v(t) = 5\,di/dt$ V.
(a) Find the charge delivered to the device between $t = 0$ and $t = 2$ s.
(b) Calculate the power absorbed.
(c) Determine the energy absorbed in 3 s.

1.13 Figure 1.24 shows the current through and the voltage across a device. Find the total energy absorbed by the device for the period of $0 < t < 4$ s.

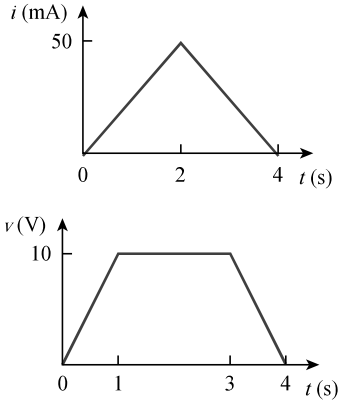


Figure 1.24 For Prob. 1.13.

Section 1.6 Circuit Elements

1.14 Figure 1.25 shows a circuit with five elements. If $p_1 = -205$ W, $p_2 = 60$ W, $p_4 = 45$ W, $p_5 = 30$ W, calculate the power p_3 received or delivered by element 3.

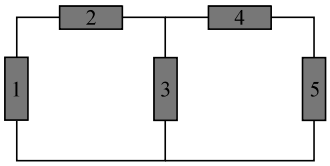


Figure 1.25 For Prob. 1.14.

1.15 Find the power absorbed by each of the elements in Fig. 1.26.

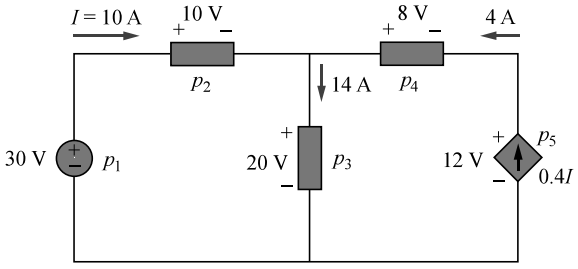


Figure 1.26 For Prob. 1.15.

1.16 Determine I_o in the circuit of Fig. 1.27.

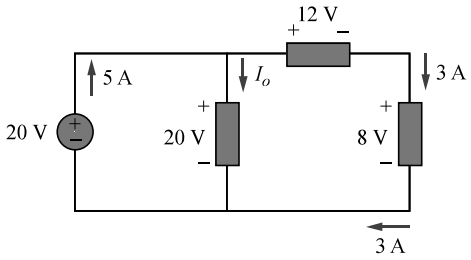


Figure 1.27 For Prob. 1.16.

1.17 Find V_o in the circuit of Fig. 1.28.

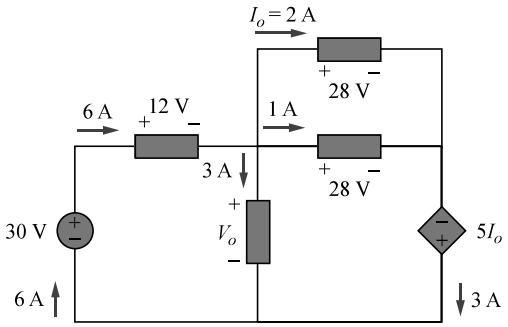


Figure 1.28 For Prob. 1.17.

Section 1.7 Applications

1.18 It takes eight photons to strike the surface of a photodetector in order to emit one electron. If 4×10^{11} photons/second strike the surface of the photodetector, calculate the amount of current flow.

1.19 Find the power rating of the following electrical appliances in your household:
(a) Lightbulb (b) Radio set
(c) TV set (d) Refrigerator
(e) Personal computer (f) PC printer
(g) Microwave oven (h) Blender

1.20 A 1.5-kW electric heater is connected to a 120-V source.
(a) How much current does the heater draw?
(b) If the heater is on for 45 minutes, how much energy is consumed in kilowatt-hours (kWh)?
(c) Calculate the cost of operating the heater for 45 minutes if energy costs 10 cents/kWh.

1.21 A 1.2-kW toaster takes roughly 4 minutes to heat four slices of bread. Find the cost of operating the toaster once per day for 1 month (30 days). Assume energy costs 9 cents/kWh.

1.22 A flashlight battery has a rating of 0.8 ampere-hours (Ah) and a lifetime of 10 hours.
(a) How much current can it deliver?
(b) How much power can it give if its terminal voltage is 6 V?
(c) How much energy is stored in the battery in kWh?

1.23 A constant current of 3 A for 4 hours is required to charge an automotive battery. If the terminal voltage is $10 + t/2$ V, where t is in hours,
(a) how much charge is transported as a result of the charging?
(b) how much energy is expended?
(c) how much does the charging cost? Assume electricity costs 9 cents/kWh.

1.24 A 30-W incandescent lamp is connected to a 120-V source and is left burning continuously in an otherwise dark staircase. Determine:
(a) the current through the lamp,
(b) the cost of operating the light for one non-leap year if electricity costs 12 cents per kWh.

1.25 An electric stove with four burners and an oven is used in preparing a meal as follows.
Burner 1: 20 minutes Burner 2: 40 minutes
Burner 3: 15 minutes Burner 4: 45 minutes
Oven: 30 minutes
If each burner is rated at 1.2 kW and the oven at 1.8 kW, and electricity costs 12 cents per kWh, calculate the cost of electricity used in preparing the meal.

1.26 PECO (the electric power company in Philadelphia) charged a consumer \$34.24 one month for using 215 kWh. If the basic service charge is \$5.10, how much did PECO charge per kWh?

COMPREHENSIVE PROBLEMS

1.27 A telephone wire has a current of 20 μA flowing through it. How long does it take for a charge of 15 C to pass through the wire?

1.28 A lightning bolt carried a current of 2 kA and lasted for 3 ms. How many coulombs of charge were contained in the lightning bolt?

1.29 The power consumption for a certain household for a day is shown in Fig. 1.29. Determine:
(a) the total energy consumed in kWh
(b) the average power per hour.

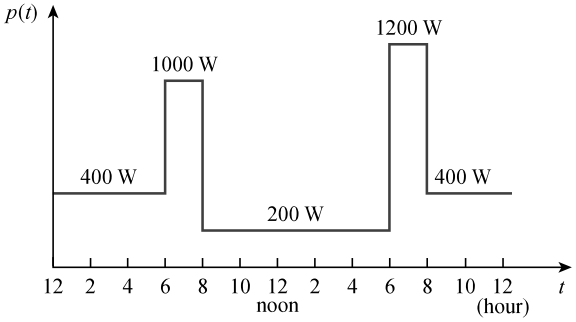


Figure 1.29 For Prob. 1.29.

1.30 The graph in Fig. 1.30 represents the power drawn by an industrial plant between 8:00 and 8:30 A.M. Calculate the total energy in MWh consumed by the plant.

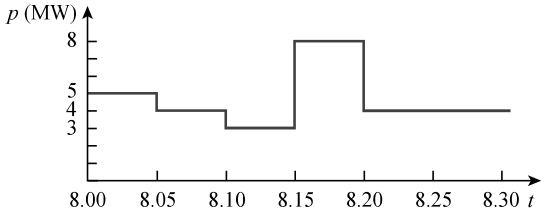


Figure 1.30 For Prob. 1.30.

1.31 A battery may be rated in ampere-hours (Ah). An lead-acid battery is rated at 160 Ah.
(a) What is the maximum current it can supply for 40 h?
(b) How many days will it last if it is discharged at 1 mA?

1.32 How much work is done by a 12-V automobile battery in moving 5×10^{20} electrons from the positive terminal to the negative terminal?

1.33 How much energy does a 10-hp motor deliver in 30 minutes? Assume that 1 horsepower = 746 W.

1.34 A 2-kW electric iron is connected to a 120-V line. Calculate the current drawn by the iron.

Go to the Student OLC

2.1 INTRODUCTION

Chapter 1 introduced basic concepts such as current, voltage, and power in an electric circuit. To actually determine the values of these variables in a given circuit requires that we understand some fundamental laws that govern electric circuits. These laws, known as Ohm's law and Kirchhoff's laws, form the foundation upon which electric circuit analysis is built.

In this chapter, in addition to these laws, we shall discuss some techniques commonly applied in circuit design and analysis. These techniques include combining resistors in series or parallel, voltage division, current division, and delta-to-wye and wye-to-delta transformations. The application of these laws and techniques will be restricted to resistive circuits in this chapter. We will finally apply the laws and techniques to real-life problems of electrical lighting and the design of dc meters.

2.2 OHM'S LAW

Materials in general have a characteristic behavior of resisting the flow of electric charge. This physical property, or ability to resist current, is known as *resistance* and is represented by the symbol R. The resistance of any material with a uniform cross-sectional area A depends on A and its length ℓ, as shown in Fig. 2.1(a). In mathematical form,

$$R = \rho \frac{\ell}{A} \tag{2.1}$$

where ρ is known as the *resistivity* of the material in ohm-meters. Good conductors, such as copper and aluminum, have low resistivities, while insulators, such as mica and paper, have high resistivities. Table 2.1 presents the values of ρ for some common materials and shows which materials are used for conductors, insulators, and semiconductors.

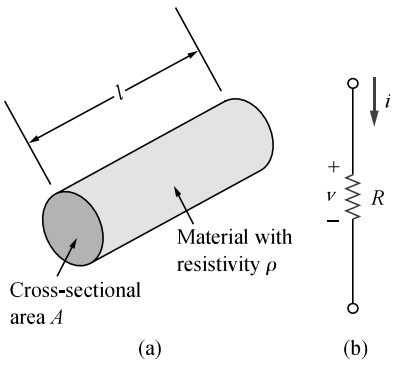


Figure 2.1 (a) Resistor, (b) Circuit symbol for resistance.

TABLE 2.1 Resistivities of common materials.

Material	Resistivity (Ω·m)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4×10^{-5}	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^{2}	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator

The circuit element used to model the current-resisting behavior of a material is the *resistor*. For the purpose of constructing circuits, resistors are usually made from metallic alloys and carbon compounds. The circuit

symbol for the resistor is shown in Fig. 2.1(b), where R stands for the resistance of the resistor. The resistor is the simplest passive element.

Georg Simon Ohm (1787–1854), a German physicist, is credited with finding the relationship between current and voltage for a resistor. This relationship is known as *Ohm's law*.

Ohm's law states that the voltage v across a resistor is directly proportional to the current i flowing through the resistor.

That is,

$$v \propto i \tag{2.2}$$

Ohm defined the constant of proportionality for a resistor to be the resistance, R. (The resistance is a material property which can change if the internal or external conditions of the element are altered, e.g., if there are changes in the temperature.) Thus, Eq. (2.2) becomes

$$\boxed{v = iR} \tag{2.3}$$

which is the mathematical form of Ohm's law. R in Eq. (2.3) is measured in the unit of ohms, designated Ω. Thus,

The resistance R of an element denotes its ability to resist the flow of electric current; it is measured in ohms (Ω).

We may deduce from Eq. (2.3) that

$$R = \frac{v}{i} \tag{2.4}$$

so that

$$1\ \Omega = 1\ \text{V/A}$$

To apply Ohm's law as stated in Eq. (2.3), we must pay careful attention to the current direction and voltage polarity. The direction of current i and the polarity of voltage v must conform with the passive sign convention, as shown in Fig. 2.1(b). This implies that current flows from a higher potential to a lower potential in order for $v = iR$. If current flows from a lower potential to a higher potential, $v = -iR$.

Since the value of R can range from zero to infinity, it is important that we consider the two extreme possible values of R. An element with $R = 0$ is called a *short circuit*, as shown in Fig. 2.2(a). For a short circuit,

$$v = iR = 0 \tag{2.5}$$

showing that the voltage is zero but the current could be anything. In practice, a short circuit is usually a connecting wire assumed to be a perfect conductor. Thus,

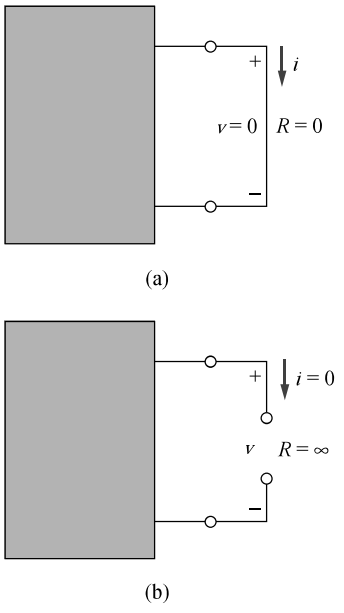


Figure 2.2 (a) Short circuit ($R = 0$), (b) Open circuit ($R = \infty$).

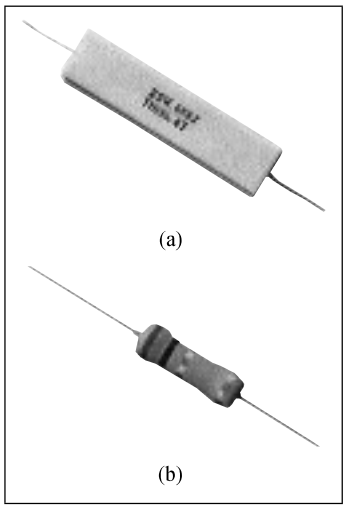


Figure 2.3 Fixed resistors: (a) wirewound type, (b) carbon film type. *(Courtesy of Tech America.)*

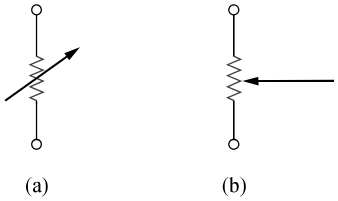


Figure 2.4 Circuit symbol for: (a) a variable resistor in general, (b) a potentiometer.

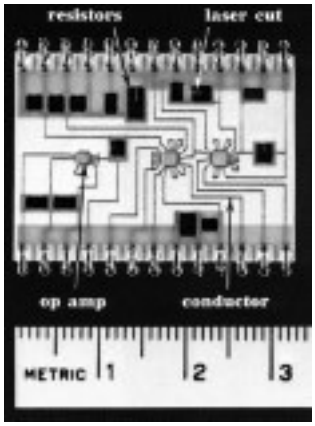


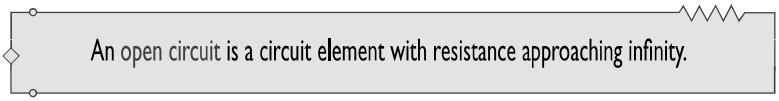
Figure 2.6 Resistors in a thick-film circuit. *(Source: G. Daryanani, Principles of Active Network Synthesis and Design [New York: John Wiley, 1976], p. 461c.)*



Similarly, an element with $R = \infty$ is known as an *open circuit*, as shown in Fig. 2.2(b). For an open circuit,

$$i = \lim_{R \to \infty} \frac{v}{R} = 0 \tag{2.6}$$

indicating that the current is zero though the voltage could be anything. Thus,



A resistor is either fixed or variable. Most resistors are of the fixed type, meaning their resistance remains constant. The two common types of fixed resistors (wirewound and composition) are shown in Fig. 2.3. The composition resistors are used when large resistance is needed. The circuit symbol in Fig. 2.1(b) is for a fixed resistor. Variable resistors have adjustable resistance. The symbol for a variable resistor is shown in Fig. 2.4(a). A common variable resistor is known as a *potentiometer* or *pot* for short, with the symbol shown in Fig. 2.4(b). The pot is a three-terminal element with a sliding contact or wiper. By sliding the wiper, the resistances between the wiper terminal and the fixed terminals vary. Like fixed resistors, variable resistors can either be of wirewound or composition type, as shown in Fig. 2.5. Although resistors like those in Figs. 2.3 and 2.5 are used in circuit designs, today most circuit components including resistors are either surface mounted or integrated, as typically shown in Fig. 2.6.

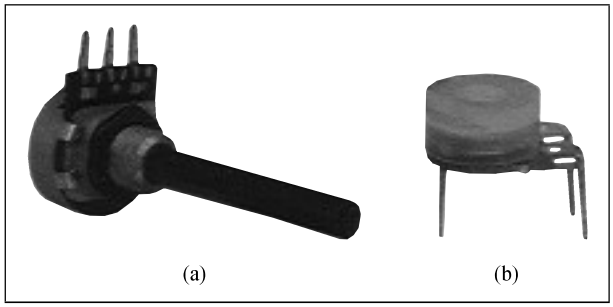


Figure 2.5 Variable resistors: (a) composition type, (b) slider pot. *(Courtesy of Tech America.)*

It should be pointed out that not all resistors obey Ohm's law. A resistor that obeys Ohm's law is known as a *linear* resistor. It has a constant resistance and thus its current-voltage characteristic is as illustrated in Fig. 2.7(a): its i-v graph is a straight line passing through the origin. A *nonlinear* resistor does not obey Ohm's law. Its resistance varies with current and its i-v characteristic is typically shown in Fig. 2.7(b).

Examples of devices with nonlinear resistance are the lightbulb and the diode. Although all practical resistors may exhibit nonlinear behavior under certain conditions, we will assume in this book that all elements actually designated as resistors are linear.

A useful quantity in circuit analysis is the reciprocal of resistance R, known as *conductance* and denoted by G:

$$G = \frac{1}{R} = \frac{i}{v} \tag{2.7}$$

The conductance is a measure of how well an element will conduct electric current. The unit of conductance is the *mho* (ohm spelled backward) or reciprocal ohm, with symbol ℧, the inverted omega. Although engineers often use the mhos, in this book we prefer to use the siemens (S), the SI unit of conductance:

$$1 \text{ S} = 1 \text{ ℧} = 1 \text{ A/V} \tag{2.8}$$

Thus,

Conductance is the ability of an element to conduct electric current; it is measured in mhos (℧) or siemens (S).

The same resistance can be expressed in ohms or siemens. For example, 10 Ω is the same as 0.1 S. From Eq. (2.7), we may write

$$i = Gv \tag{2.9}$$

The power dissipated by a resistor can be expressed in terms of R. Using Eqs. (1.7) and (2.3),

$$p = vi = i^2R = \frac{v^2}{R} \tag{2.10}$$

The power dissipated by a resistor may also be expressed in terms of G as

$$p = vi = v^2G = \frac{i^2}{G} \tag{2.11}$$

We should note two things from Eqs. (2.10) and (2.11):

1. The power dissipated in a resistor is a nonlinear function of either current or voltage.
2. Since R and G are positive quantities, the power dissipated in a resistor is always positive. Thus, a resistor always absorbs power from the circuit. This confirms the idea that a resistor is a passive element, incapable of generating energy.

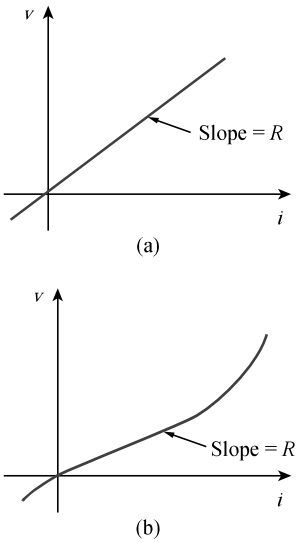


Figure 2.7 The i-v characteristic of: (a) a linear resistor, (b) a nonlinear resistor.

EXAMPLE 2.1

An electric iron draws 2 A at 120 V. Find its resistance.

Solution:

From Ohm's law,

$$R = \frac{v}{i} = \frac{120}{2} = 60\ \Omega$$

PRACTICE PROBLEM 2.1

The essential component of a toaster is an electrical element (a resistor) that converts electrical energy to heat energy. How much current is drawn by a toaster with resistance 12 Ω at 110 V?

Answer: 9.167 A.

EXAMPLE 2.2

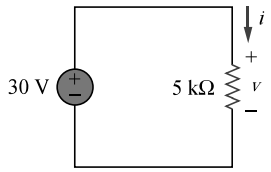


Figure 2.8 For Example 2.2.

In the circuit shown in Fig. 2.8, calculate the current i, the conductance G, and the power p.

Solution:

The voltage across the resistor is the same as the source voltage (30 V) because the resistor and the voltage source are connected to the same pair of terminals. Hence, the current is

$$i = \frac{v}{R} = \frac{30}{5 \times 10^3} = 6\ \text{mA}$$

The conductance is

$$G = \frac{1}{R} = \frac{1}{5 \times 10^3} = 0.2\ \text{mS}$$

We can calculate the power in various ways using either Eqs. (1.7), (2.10), or (2.11).

$$p = vi = 30(6 \times 10^{-3}) = 180\ \text{mW}$$

or

$$p = i^2R = (6 \times 10^{-3})^2 5 \times 10^3 = 180\ \text{mW}$$

or

$$p = v^2G = (30)^2 0.2 \times 10^{-3} = 180\ \text{mW}$$

PRACTICE PROBLEM 2.2

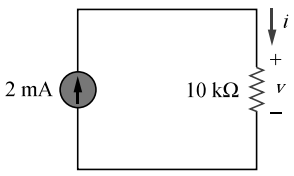


Figure 2.9 For Practice Prob. 2.2

For the circuit shown in Fig. 2.9, calculate the voltage v, the conductance G, and the power p.

Answer: 20 V, 100 μS, 40 mW.

EXAMPLE 2.3

A voltage source of $20 \sin \pi t$ V is connected across a 5-kΩ resistor. Find the current through the resistor and the power dissipated.

Solution:

$$i = \frac{v}{R} = \frac{20 \sin \pi t}{5 \times 10^3} = 4 \sin \pi t \text{ mA}$$

Hence,

$$p = vi = 80 \sin^2 \pi t \text{ mW}$$

PRACTICE PROBLEM 2.3

A resistor absorbs an instantaneous power of $20 \cos^2 t$ mW when connected to a voltage source $v = 10 \cos t$ V. Find i and R.

Answer: $2 \cos t$ mA, 5 kΩ.

†2.3 NODES, BRANCHES, AND LOOPS

Since the elements of an electric circuit can be interconnected in several ways, we need to understand some basic concepts of network topology. To differentiate between a circuit and a network, we may regard a network as an interconnection of elements or devices, whereas a circuit is a network providing one or more closed paths. The convention, when addressing network topology, is to use the word network rather than circuit. We do this even though the words network and circuit mean the same thing when used in this context. In network topology, we study the properties relating to the placement of elements in the network and the geometric configuration of the network. Such elements include branches, nodes, and loops.

A branch represents a single element such as a voltage source or a resistor.

In other words, a branch represents any two-terminal element. The circuit in Fig. 2.10 has five branches, namely, the 10-V voltage source, the 2-A current source, and the three resistors.

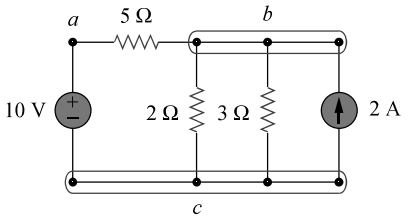


Figure 2.10 Nodes, branches, and loops.

A node is the point of connection between two or more branches.

A node is usually indicated by a dot in a circuit. If a short circuit (a connecting wire) connects two nodes, the two nodes constitute a single node. The circuit in Fig. 2.10 has three nodes a, b, and c. Notice that the three points that form node b are connected by perfectly conducting wires and therefore constitute a single point. The same is true of the four points forming node c. We demonstrate that the circuit in Fig. 2.10 has only three nodes by redrawing the circuit in Fig. 2.11. The two circuits in

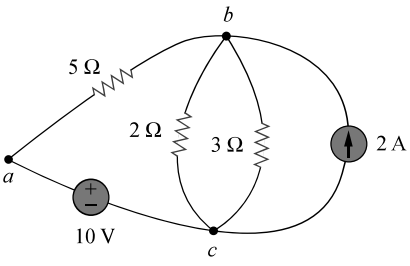


Figure 2.11 The three-node circuit of Fig. 2.10 is redrawn.

Figs. 2.10 and 2.11 are identical. However, for the sake of clarity, nodes b and c are spread out with perfect conductors as in Fig. 2.10.

A loop is any closed path in a circuit.

A loop is a closed path formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through any node more than once. A loop is said to be *independent* if it contains a branch which is not in any other loop. Independent loops or paths result in independent sets of equations.

For example, the closed path *abca* containing the 2-Ω resistor in Fig. 2.11 is a loop. Another loop is the closed path *bcb* containing the 3-Ω resistor and the current source. Although one can identify six loops in Fig. 2.11, only three of them are independent.

A network with b branches, n nodes, and l independent loops will satisfy the fundamental theorem of network topology:

$$b = l + n - 1 \tag{2.12}$$

As the next two definitions show, circuit topology is of great value to the study of voltages and currents in an electric circuit.

Two or more elements are in series if they are cascaded or connected sequentially and consequently carry the same current.
Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them.

Elements are in series when they are chain-connected or connected sequentially, end to end. For example, two elements are in series if they share one common node and no other element is connected to that common node. Elements in parallel are connected to the same pair of terminals. Elements may be connected in a way that they are neither in series nor in parallel. In the circuit shown in Fig. 2.10, the voltage source and the 5-Ω resistor are in series because the same current will flow through them. The 2-Ω resistor, the 3-Ω resistor, and the current source are in parallel because they are connected to the same two nodes (b and c) and consequently have the same voltage across them. The 5-Ω and 2-Ω resistors are neither in series nor in parallel with each other.

EXAMPLE 2.4

Determine the number of branches and nodes in the circuit shown in Fig. 2.12. Identify which elements are in series and which are in parallel.

Solution:

Since there are four elements in the circuit, the circuit has four branches: 10 V, 5 Ω, 6 Ω, and 2 A. The circuit has three nodes as identified in

Fig. 2.13. The 5-Ω resistor is in series with the 10-V voltage source because the same current would flow in both. The 6-Ω resistor is in parallel with the 2-A current source because both are connected to the same nodes 2 and 3.

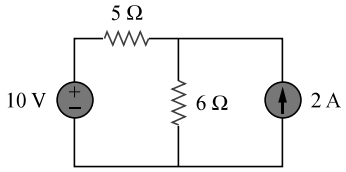


Figure 2.12 For Example 2.4.

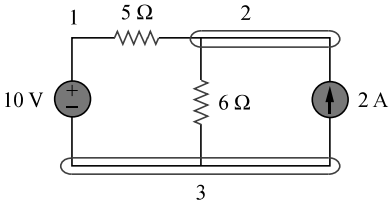


Figure 2.13 The three nodes in the circuit of Fig. 2.12.

PRACTICE PROBLEM 2.4

How many branches and nodes does the circuit in Fig. 2.14 have? Identify the elements that are in series and in parallel.

Answer: Five branches and three nodes are identified in Fig. 2.15. The 1-Ω and 2-Ω resistors are in parallel. The 4-Ω resistor and 10-V source are also in parallel.

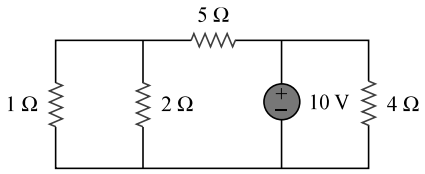


Figure 2.14 For Practice Prob. 2.4.

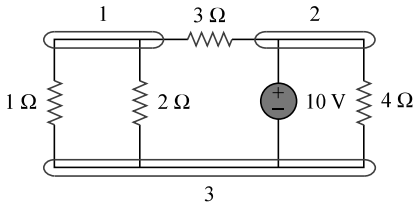


Figure 2.15 Answer for Practice Prob. 2.4.

2.4 KIRCHHOFF'S LAWS

Ohm's law by itself is not sufficient to analyze circuits. However, when it is coupled with Kirchhoff's two laws, we have a sufficient, powerful set of tools for analyzing a large variety of electric circuits. Kirchhoff's laws were first introduced in 1847 by the German physicist Gustav Robert Kirchhoff (1824–1887). These laws are formally known as Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL).

Kirchhoff's first law is based on the law of conservation of charge, which requires that the algebraic sum of charges within a system cannot change.

Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.

Mathematically, KCL implies that

$$\sum_{n=1}^{N} i_n = 0 \tag{2.13}$$

where N is the number of branches connected to the node and i_n is the nth current entering (or leaving) the node. By this law, currents entering a node may be regarded as positive, while currents leaving the node may be taken as negative or vice versa.

To prove KCL, assume a set of currents $i_k(t), k = 1, 2, \ldots,$ flow into a node. The algebraic sum of currents at the node is

$$i_T(t) = i_1(t) + i_2(t) + i_3(t) + \cdots \tag{2.14}$$

Integrating both sides of Eq. (2.14) gives

$$q_T(t) = q_1(t) + q_2(t) + q_3(t) + \cdots \tag{2.15}$$

where $q_k(t) = \int i_k(t)\, dt$ and $q_T(t) = \int i_T(t)\, dt$. But the law of conservation of electric charge requires that the algebraic sum of electric charges at the node must not change; that is, the node stores no net charge. Thus $q_T(t) = 0 \rightarrow i_T(t) = 0$, confirming the validity of KCL.

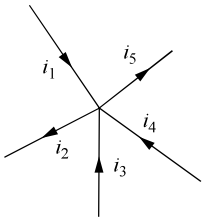


Figure 2.16 Currents at a node illustrating KCL.

Consider the node in Fig. 2.16. Applying KCL gives

$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0 \tag{2.16}$$

since currents i_1, i_3, and i_4 are entering the node, while currents i_2 and i_5 are leaving it. By rearranging the terms, we get

$$i_1 + i_3 + i_4 = i_2 + i_5 \tag{2.17}$$

Equation (2.17) is an alternative form of KCL:

The sum of the currents entering a node is equal to the sum of the currents leaving the node.

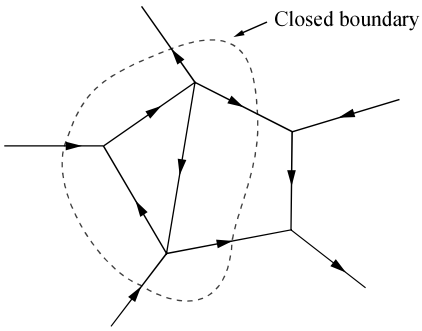


Figure 2.17 Applying KCL to a closed boundary.

Note that KCL also applies to a closed boundary. This may be regarded as a generalized case, because a node may be regarded as a closed surface shrunk to a point. In two dimensions, a closed boundary is the same as a closed path. As typically illustrated in the circuit of Fig. 2.17, the total current entering the closed surface is equal to the total current leaving the surface.

A simple application of KCL is combining current sources in parallel. The combined current is the algebraic sum of the current supplied by the individual sources. For example, the current sources shown in Fig. 2.18(a) can be combined as in Fig. 2.18(b). The combined or equivalent current source can be found by applying KCL to node a.

$$I_T + I_2 = I_1 + I_3$$

Two sources (or circuits in general) are said to be equivalent if they have the same i-v relationship at a pair of terminals.

or

$$I_T = I_1 - I_2 + I_3 \tag{2.18}$$

A circuit cannot contain two different currents, I_1 and I_2, in series, unless $I_1 = I_2$; otherwise KCL will be violated.

Kirchhoff's second law is based on the principle of conservation of energy:

Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.

Expressed mathematically, KVL states that

$$\sum_{m=1}^{M} v_m = 0 \tag{2.19}$$

where M is the number of voltages in the loop (or the number of branches in the loop) and v_m is the mth voltage.

To illustrate KVL, consider the circuit in Fig. 2.19. The sign on each voltage is the polarity of the terminal encountered first as we travel around the loop. We can start with any branch and go around the loop either clockwise or counterclockwise. Suppose we start with the voltage source and go clockwise around the loop as shown; then voltages would be $-v_1, +v_2, +v_3, -v_4$, and $+v_5$, in that order. For example, as we reach branch 3, the positive terminal is met first; hence we have $+v_3$. For branch 4, we reach the negative terminal first; hence, $-v_4$. Thus, KVL yields

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0 \tag{2.20}$$

Rearranging terms gives

$$v_2 + v_3 + v_5 = v_1 + v_4 \tag{2.21}$$

which may be interpreted as

$$\text{Sum of voltage drops} = \text{Sum of voltage rises} \tag{2.22}$$

This is an alternative form of KVL. Notice that if we had traveled counterclockwise, the result would have been $+v_1, -v_5, +v_4, -v_3$, and $-v_2$, which is the same as before except that the signs are reversed. Hence, Eqs. (2.20) and (2.21) remain the same.

When voltage sources are connected in series, KVL can be applied to obtain the total voltage. The combined voltage is the algebraic sum of the voltages of the individual sources. For example, for the voltage sources shown in Fig. 2.20(a), the combined or equivalent voltage source in Fig. 2.20(b) is obtained by applying KVL.

$$-V_{ab} + V_1 + V_2 - V_3 = 0$$

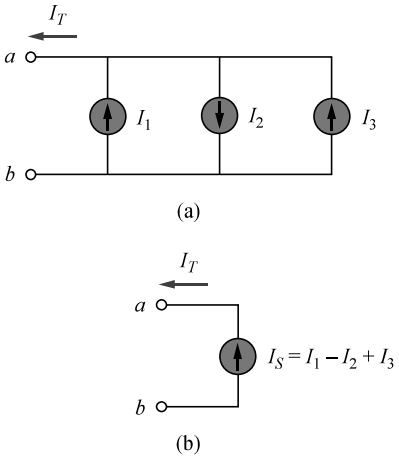


Figure 2.18 Current sources in parallel: (a) original circuit, (b) equivalent circuit.

KVL can be applied in two ways: by taking either a clockwise or a counterclockwise trip around the loop. Either way, the algebraic sum of voltages around the loop is zero.

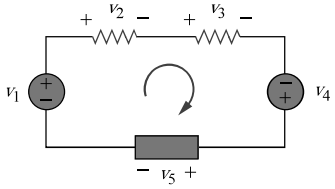


Figure 2.19 A single-loop circuit illustrating KVL.

or

$$V_{ab} = V_1 + V_2 - V_3 \tag{2.23}$$

To avoid violating KVL, a circuit cannot contain two different voltages V_1 and V_2 in parallel unless $V_1 = V_2$.

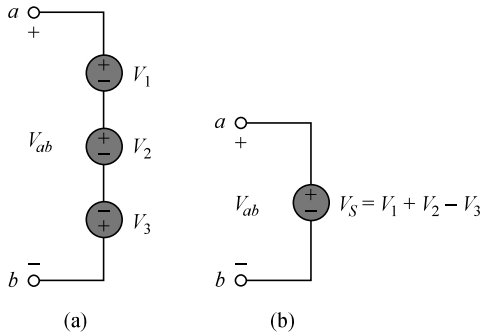


Figure 2.20 Voltage sources in series: (a) original circuit, (b) equivalent circuit.

EXAMPLE 2.5

For the circuit in Fig. 2.21(a), find voltages v_1 and v_2.

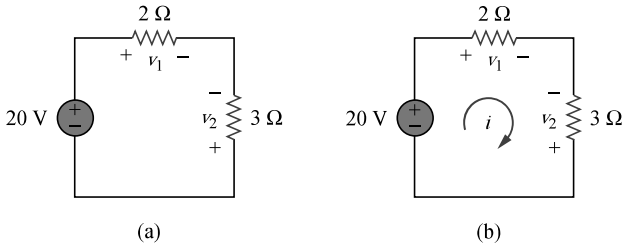


Figure 2.21 For Example 2.5.

Solution:

To find v_1 and v_2, we apply Ohm's law and Kirchhoff's voltage law. Assume that current i flows through the loop as shown in Fig. 2.21(b). From Ohm's law,

$$v_1 = 2i, \qquad v_2 = -3i \tag{2.5.1}$$

Applying KVL around the loop gives

$$-20 + v_1 - v_2 = 0 \tag{2.5.2}$$

Substituting Eq. (2.5.1) into Eq. (2.5.2), we obtain

$$-20 + 2i + 3i = 0 \quad \text{or} \quad 5i = 20 \quad \Longrightarrow \quad i = 4 \text{ A}$$

Substituting i in Eq. (2.5.1) finally gives

$$v_1 = 8 \text{ V}, \qquad v_2 = -12 \text{ V}$$

PRACTICE PROBLEM 2.5

Find v_1 and v_2 in the circuit of Fig. 2.22.

Answer: 12 V, −6 V.

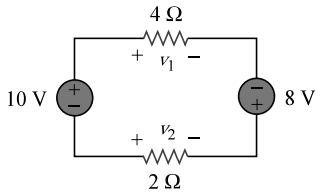


Figure 2.22 For Practice Prob. 2.5

EXAMPLE 2.6

Determine v_o and i in the circuit shown in Fig. 2.23(a).

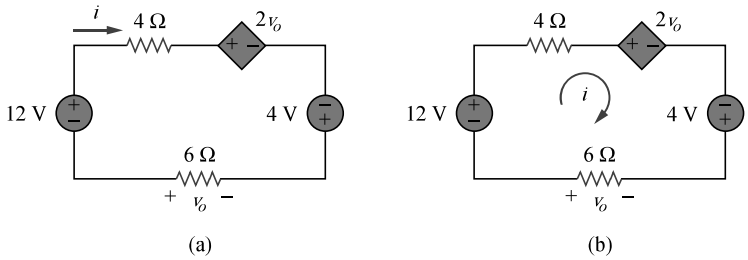


Figure 2.23 For Example 2.6.

Solution:

We apply KVL around the loop as shown in Fig. 2.23(b). The result is

$$-12 + 4i + 2v_o - 4 + 6i = 0 \tag{2.6.1}$$

Applying Ohm's law to the 6-Ω resistor gives

$$v_o = -6i \tag{2.6.2}$$

Substituting Eq. (2.6.2) into Eq. (2.6.1) yields

$$-16 + 10i - 12i = 0 \quad \Longrightarrow \quad i = -8 \text{ A}$$

and $v_o = 48$ V.

PRACTICE PROBLEM 2.6

Find v_x and v_o in the circuit of Fig. 2.24.

Answer: 10 V, −5 V.

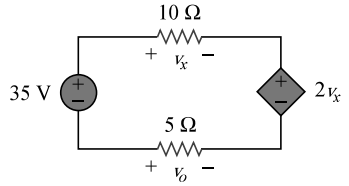


Figure 2.24 For Practice Prob. 2.6.

EXAMPLE 2.7

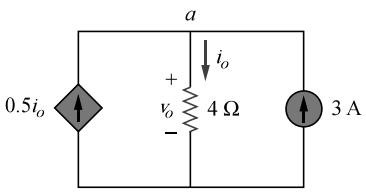


Figure 2.25 For Example 2.7.

Find current i_o and voltage v_o in the circuit shown in Fig. 2.25.

Solution:

Applying KCL to node a, we obtain

$$3 + 0.5i_o = i_o \qquad \Longrightarrow \qquad i_o = 6 \text{ A}$$

For the 4-Ω resistor, Ohm's law gives

$$v_o = 4i_o = 24 \text{ V}$$

PRACTICE PROBLEM 2.7

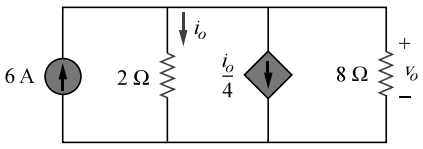


Figure 2.26 For Practice Prob. 2.7.

Find v_o and i_o in the circuit of Fig. 2.26.

Answer: 8 V, 4 A.

EXAMPLE 2.8

Find the currents and voltages in the circuit shown in Fig. 2.27(a).

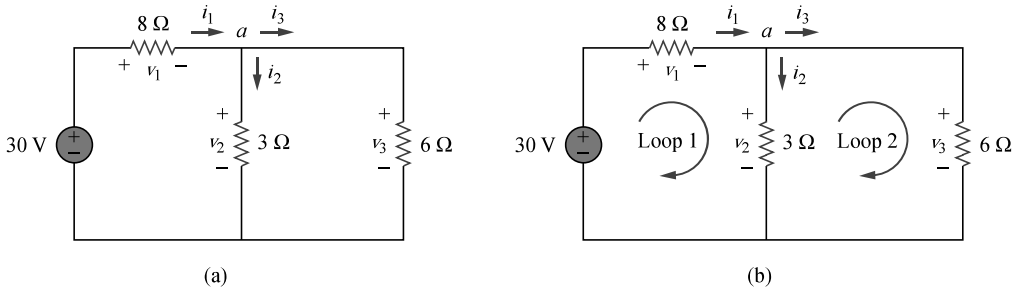


Figure 2.27 For Example 2.8.

Solution:

We apply Ohm's law and Kirchhoff's laws. By Ohm's law,

$$v_1 = 8i_1, \qquad v_2 = 3i_2, \qquad v_3 = 6i_3 \tag{2.8.1}$$

Since the voltage and current of each resistor are related by Ohm's law as shown, we are really looking for three things: (v_1, v_2, v_3) or (i_1, i_2, i_3). At node a, KCL gives

$$i_1 - i_2 - i_3 = 0 \tag{2.8.2}$$

Applying KVL to loop 1 as in Fig. 2.27(b),

$$-30 + v_1 + v_2 = 0$$

We express this in terms of i_1 and i_2 as in Eq. (2.8.1) to obtain

$$-30 + 8i_1 + 3i_2 = 0$$

or

$$i_1 = \frac{(30 - 3i_2)}{8} \tag{2.8.3}$$

Applying KVL to loop 2,

$$-v_2 + v_3 = 0 \qquad \Longrightarrow \qquad v_3 = v_2 \tag{2.8.4}$$

as expected since the two resistors are in parallel. We express v_1 and v_2 in terms of i_1 and i_2 as in Eq. (2.8.1). Equation (2.8.4) becomes

$$6i_3 = 3i_2 \qquad \Longrightarrow \qquad i_3 = \frac{i_2}{2} \tag{2.8.5}$$

Substituting Eqs. (2.8.3) and (2.8.5) into (2.8.2) gives

$$\frac{30 - 3i_2}{8} - i_2 - \frac{i_2}{2} = 0$$

or $i_2 = 2$ A. From the value of i_2, we now use Eqs. (2.8.1) to (2.8.5) to obtain

$$i_1 = 3 \text{ A}, \qquad i_3 = 1 \text{ A}, \qquad v_1 = 24 \text{ V}, \qquad v_2 = 6 \text{ V}, \qquad v_3 = 6 \text{ V}$$



PRACTICE PROBLEM 2.8

Find the currents and voltages in the circuit shown in Fig. 2.28.

Answer: $v_1 = 3$ V, $v_2 = 2$ V, $v_3 = 5$ V, $i_1 = 1.5$ A, $i_2 = 0.25$ A, $i_3 = 1.25$ A.

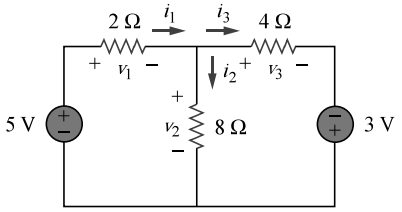


Figure 2.28 For Practice Prob. 2.8.

2.5 SERIES RESISTORS AND VOLTAGE DIVISION

The need to combine resistors in series or in parallel occurs so frequently that it warrants special attention. The process of combining the resistors is facilitated by combining two of them at a time. With this in mind, consider the single-loop circuit of Fig. 2.29. The two resistors are in series, since the same current i flows in both of them. Applying Ohm's law to each of the resistors, we obtain

$$v_1 = iR_1, \qquad v_2 = iR_2 \tag{2.24}$$

If we apply KVL to the loop (moving in the clockwise direction), we have

$$-v + v_1 + v_2 = 0 \tag{2.25}$$

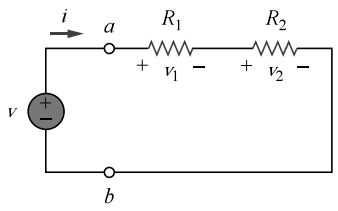


Figure 2.29 A single-loop circuit with two resistors in series.

Combining Eqs. (2.24) and (2.25), we get

$$v = v_1 + v_2 = i(R_1 + R_2) \tag{2.26}$$

or

$$i = \frac{v}{R_1 + R_2} \tag{2.27}$$

Notice that Eq. (2.26) can be written as

$$v = iR_{\text{eq}} \tag{2.28}$$

implying that the two resistors can be replaced by an equivalent resistor R_{eq}; that is,

$$R_{\text{eq}} = R_1 + R_2 \tag{2.29}$$

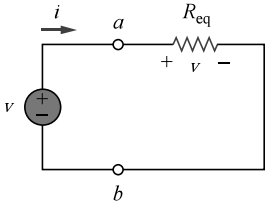


Figure 2.30 Equivalent circuit of the Fig. 2.29 circuit.

Thus, Fig. 2.29 can be replaced by the equivalent circuit in Fig. 2.30. The two circuits in Figs. 2.29 and 2.30 are equivalent because they exhibit the same voltage-current relationships at the terminals a-b. An equivalent circuit such as the one in Fig. 2.30 is useful in simplifying the analysis of a circuit. In general,

The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.

Resistors in series behave as a single resistor whose resistance is equal to the sum of the resistances of the individual resistors.

For N resistors in series then,

$$R_{\text{eq}} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n \tag{2.30}$$

To determine the voltage across each resistor in Fig. 2.29, we substitute Eq. (2.26) into Eq. (2.24) and obtain

$$v_1 = \frac{R_1}{R_1 + R_2} v, \qquad v_2 = \frac{R_2}{R_1 + R_2} v \tag{2.31}$$

Notice that the source voltage v is divided among the resistors in direct proportion to their resistances; the larger the resistance, the larger the voltage drop. This is called the *principle of voltage division*, and the circuit in Fig. 2.29 is called a *voltage divider*. In general, if a voltage divider has N resistors ($R_1, R_2, \ldots, R_N$) in series with the source voltage v, the nth resistor (R_n) will have a voltage drop of

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v \tag{2.32}$$

2.6 PARALLEL RESISTORS AND CURRENT DIVISION

Consider the circuit in Fig. 2.31, where two resistors are connected in parallel and therefore have the same voltage across them. From Ohm's law,

$$v = i_1 R_1 = i_2 R_2$$

or

$$i_1 = \frac{v}{R_1}, \qquad i_2 = \frac{v}{R_2} \tag{2.33}$$

Applying KCL at node a gives the total current i as

$$i = i_1 + i_2 \tag{2.34}$$

Substituting Eq. (2.33) into Eq. (2.34), we get

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{\text{eq}}} \tag{2.35}$$

where R_{eq} is the equivalent resistance of the resistors in parallel:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} \tag{2.36}$$

or

$$\frac{1}{R_{\text{eq}}} = \frac{R_1 + R_2}{R_1 R_2}$$

or

$$\boxed{R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2}} \tag{2.37}$$

Thus,

The equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum.

It must be emphasized that this applies only to two resistors in parallel. From Eq. (2.37), if $R_1 = R_2$, then $R_{\text{eq}} = R_1/2$.

We can extend the result in Eq. (2.36) to the general case of a circuit with N resistors in parallel. The equivalent resistance is

$$\boxed{\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}} \tag{2.38}$$

Note that R_{eq} is always smaller than the resistance of the smallest resistor in the parallel combination. If $R_1 = R_2 = \cdots = R_N = R$, then

$$R_{\text{eq}} = \frac{R}{N} \tag{2.39}$$

For example, if four 100-Ω resistors are connected in parallel, their equivalent resistance is 25 Ω.

It is often more convenient to use conductance rather than resistance when dealing with resistors in parallel. From Eq. (2.38), the equivalent conductance for N resistors in parallel is

$$\boxed{G_{\text{eq}} = G_1 + G_2 + G_3 + \cdots + G_N} \tag{2.40}$$

where $G_{\text{eq}} = 1/R_{\text{eq}}$, $G_1 = 1/R_1$, $G_2 = 1/R_2$, $G_3 = 1/R_3, \ldots, G_N = 1/R_N$. Equation (2.40) states:

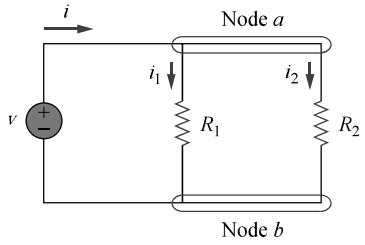


Figure 2.31 Two resistors in parallel.

Conductances in parallel behave as a single conductance whose value is equal to the sum of the individual conductances.

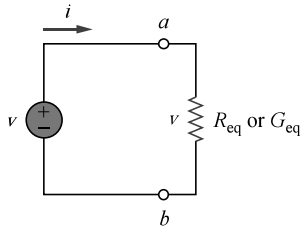


Figure 2.32 Equivalent circuit to Fig. 2.31.

The equivalent conductance of resistors connected in parallel is the sum of their individual conductances.

This means that we may replace the circuit in Fig. 2.31 with that in Fig. 2.32. Notice the similarity between Eqs. (2.30) and (2.40). The equivalent conductance of parallel resistors is obtained the same way as the equivalent resistance of series resistors. In the same manner, the equivalent conductance of resistors in series is obtained just the same way as the resistance of resistors in parallel. Thus the equivalent conductance G_{eq} of N resistors in series (such as shown in Fig. 2.29) is

$$\boxed{\frac{1}{G_{eq}} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3} + \cdots + \frac{1}{G_N}} \tag{2.41}$$

Given the total current i entering node a in Fig. 2.31, how do we obtain current i_1 and i_2? We know that the equivalent resistor has the same voltage, or

$$v = iR_{eq} = \frac{iR_1R_2}{R_1 + R_2} \tag{2.42}$$

Combining Eqs. (2.33) and (2.42) results in

$$\boxed{i_1 = \frac{R_2\, i}{R_1 + R_2}, \qquad i_2 = \frac{R_1\, i}{R_1 + R_2}} \tag{2.43}$$

which shows that the total current i is shared by the resistors in inverse proportion to their resistances. This is known as the *principle of current division*, and the circuit in Fig. 2.31 is known as a *current divider*. Notice that the larger current flows through the smaller resistance.

As an extreme case, suppose one of the resistors in Fig. 2.31 is zero, say $R_2 = 0$; that is, R_2 is a short circuit, as shown in Fig. 2.33(a). From Eq. (2.43), $R_2 = 0$ implies that $i_1 = 0$, $i_2 = i$. This means that the entire current i bypasses R_1 and flows through the short circuit $R_2 = 0$, the path of least resistance. Thus when a circuit is short circuited, as shown in Fig. 2.33(a), two things should be kept in mind:

1. The equivalent resistance $R_{eq} = 0$. [See what happens when $R_2 = 0$ in Eq. (2.37).]
2. The entire current flows through the short circuit.

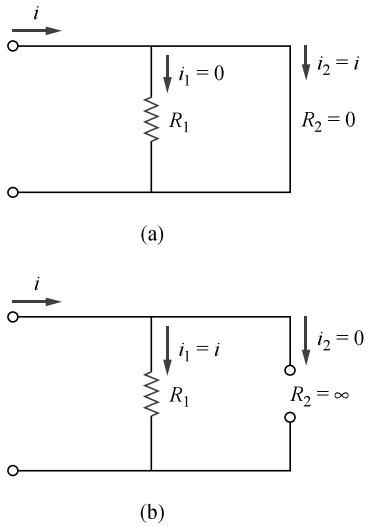


Figure 2.33 (a) A shorted circuit, (b) an open circuit.

As another extreme case, suppose $R_2 = \infty$, that is, R_2 is an open circuit, as shown in Fig. 2.33(b). The current still flows through the path of least resistance, R_1. By taking the limit of Eq. (2.37) as $R_2 \to \infty$, we obtain $R_{eq} = R_1$ in this case.

If we divide both the numerator and denominator by R_1R_2, Eq. (2.43) becomes

$$i_1 = \frac{G_1}{G_1 + G_2} i \tag{2.44a}$$

$$i_2 = \frac{G_2}{G_1 + G_2} i \tag{2.44b}$$

Thus, in general, if a current divider has N conductors ($G_1, G_2, \ldots, G_N$) in parallel with the source current i, the nth conductor (G_n) will have current

$$i_n = \frac{G_n}{G_1 + G_2 + \cdots + G_N} i \tag{2.45}$$

In general, it is often convenient and possible to combine resistors in series and parallel and reduce a resistive network to a single *equivalent resistance* R_{eq}. Such an equivalent resistance is the resistance between the designated terminals of the network and must exhibit the same i-v characteristics as the original network at the terminals.

EXAMPLE 2.9

Find R_{eq} for the circuit shown in Fig. 2.34.

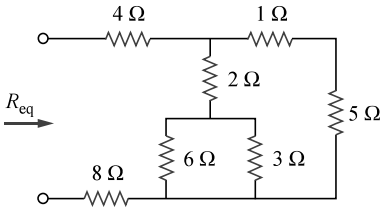


Figure 2.34 For Example 2.9.

Solution:

To get R_{eq}, we combine resistors in series and in parallel. The 6-Ω and 3-Ω resistors are in parallel, so their equivalent resistance is

$$6\ \Omega \parallel 3\ \Omega = \frac{6 \times 3}{6 + 3} = 2\ \Omega$$

(The symbol $\parallel$ is used to indicate a parallel combination.) Also, the 1-Ω and 5-Ω resistors are in series; hence their equivalent resistance is

$$1\ \Omega + 5\ \Omega = 6\ \Omega$$

Thus the circuit in Fig. 2.34 is reduced to that in Fig. 2.35(a). In Fig. 2.35(a), we notice that the two 2-Ω resistors are in series, so the equivalent resistance is

$$2\ \Omega + 2\ \Omega = 4\ \Omega$$

This 4-Ω resistor is now in parallel with the 6-Ω resistor in Fig. 2.35(a); their equivalent resistance is

$$4\ \Omega \parallel 6\ \Omega = \frac{4 \times 6}{4 + 6} = 2.4\ \Omega$$

The circuit in Fig. 2.35(a) is now replaced with that in Fig. 2.35(b). In Fig. 2.35(b), the three resistors are in series. Hence, the equivalent resistance for the circuit is

$$R_{eq} = 4\ \Omega + 2.4\ \Omega + 8\ \Omega = 14.4\ \Omega$$

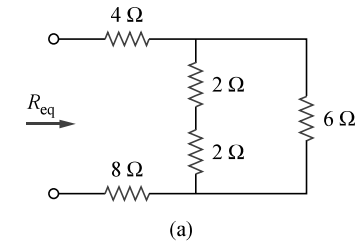


Figure 2.35 Equivalent circuits for Example 2.9.

PRACTICE PROBLEM 2.9

By combining the resistors in Fig. 2.36, find R_{eq}.

Answer: 6 Ω.

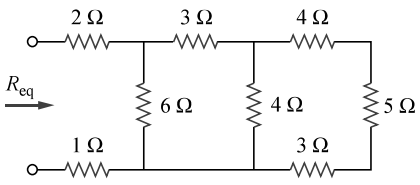


Figure 2.36 For Practice Prob. 2.9.

EXAMPLE 2.10

Calculate the equivalent resistance R_{ab} in the circuit in Fig. 2.37.

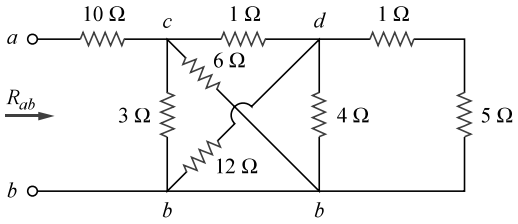


Figure 2.37 For Example 2.10.

Solution:

The 3-Ω and 6-Ω resistors are in parallel because they are connected to the same two nodes c and b. Their combined resistance is

$$3\ \Omega \parallel 6\ \Omega = \frac{3 \times 6}{3+6} = 2\ \Omega \tag{2.10.1}$$

Similarly, the 12-Ω and 4-Ω resistors are in parallel since they are connected to the same two nodes d and b. Hence

$$12\ \Omega \parallel 4\ \Omega = \frac{12 \times 4}{12+4} = 3\ \Omega \tag{2.10.2}$$

Also the 1-Ω and 5-Ω resistors are in series; hence, their equivalent resistance is

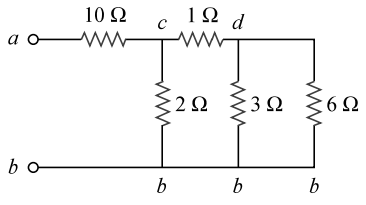
$$1\ \Omega + 5\ \Omega = 6\ \Omega \tag{2.10.3}$$

With these three combinations, we can replace the circuit in Fig. 2.37 with that in Fig. 2.38(a). In Fig. 2.38(a), 3-Ω in parallel with 6-Ω gives 2-Ω, as calculated in Eq. (2.10.1). This 2-Ω equivalent resistance is now in series with the 1-Ω resistance to give a combined resistance of 1 Ω+2 Ω = 3 Ω. Thus, we replace the circuit in Fig. 2.38(a) with that in Fig. 2.38(b). In Fig. 2.38(b), we combine the 2-Ω and 3-Ω resistors in parallel to get

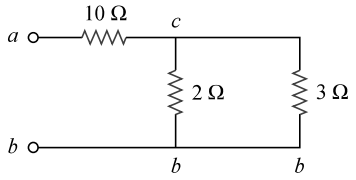
$$2\ \Omega \parallel 3\ \Omega = \frac{2 \times 3}{2+3} = 1.2\ \Omega$$

This 1.2-Ω resistor is in series with the 10-Ω resistor, so that

$$R_{ab} = 10 + 1.2 = 11.2\ \Omega$$



(a)



(b)

Figure 2.38 Equivalent circuits for Example 2.10.

PRACTICE PROBLEM 2.10

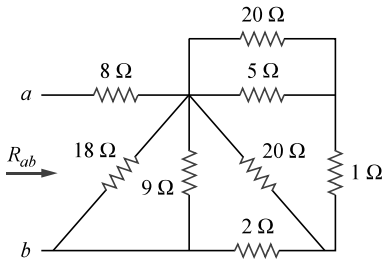


Figure 2.39 For Practice Prob. 2.10.

Find R_{ab} for the circuit in Fig. 2.39.

Answer: 11 Ω.

EXAMPLE 2.11

Find the equivalent conductance G_{eq} for the circuit in Fig. 2.40(a).

Solution:

The 8-S and 12-S resistors are in parallel, so their conductance is

$$8\ \text{S} + 12\ \text{S} = 20\ \text{S}$$

This 20-S resistor is now in series with 5 S as shown in Fig. 2.40(b) so that the combined conductance is

$$\frac{20 \times 5}{20 + 5} = 4\ \text{S}$$

This is in parallel with the 6-S resistor. Hence

$$G_{eq} = 6 + 4 = 10\ \text{S}$$

We should note that the circuit in Fig. 2.40(a) is the same as that in Fig. 2.40(c). While the resistors in Fig. 2.40(a) are expressed in siemens, they are expressed in ohms in Fig. 2.40(c). To show that the circuits are the same, we find R_{eq} for the circuit in Fig. 2.40(c).

$$R_{eq} = \frac{1}{6} \Big\| \left(\frac{1}{5} + \frac{1}{8} \Big\| \frac{1}{12} \right) = \frac{1}{6} \Big\| \left(\frac{1}{5} + \frac{1}{20} \right) = \frac{1}{6} \Big\| \frac{1}{4}$$

$$= \frac{\frac{1}{6} \times \frac{1}{4}}{\frac{1}{6} + \frac{1}{4}} = \frac{1}{10}\ \Omega$$

$$G_{eq} = \frac{1}{R_{eq}} = 10\ \text{S}$$

This is the same as we obtained previously.

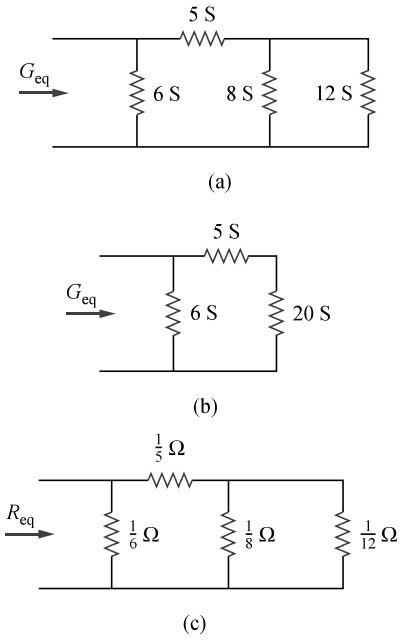


Figure 2.40 For Example 2.11: (a) original circuit, (b) its equivalent circuit, (c) same circuit as in (a) but resistors are expressed in ohms.

PRACTICE PROBLEM 2.11

Calculate G_{eq} in the circuit of Fig. 2.41.

Answer: 4 S.

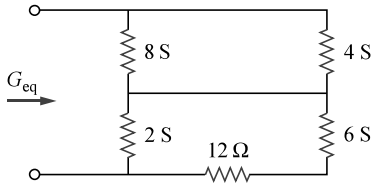


Figure 2.41 For Practice Prob. 2.11.

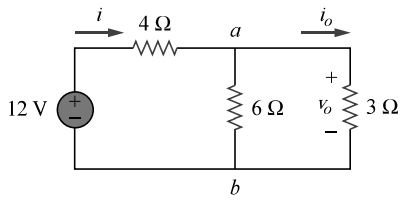
EXAMPLE 2.12

Find i_o and v_o in the circuit shown in Fig. 2.42(a). Calculate the power dissipated in the 3-Ω resistor.

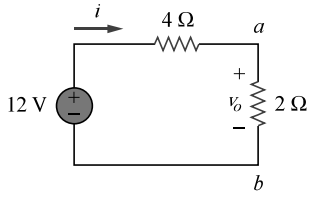
Solution:

The 6-Ω and 3-Ω resistors are in parallel, so their combined resistance is

$$6\ \Omega \parallel 3\ \Omega = \frac{6 \times 3}{6 + 3} = 2\ \Omega$$



(a)



(b)

Figure 2.42 For Example 2.12: (a) original circuit, (b) its equivalent circuit.

Thus our circuit reduces to that shown in Fig. 2.42(b). Notice that v_o is not affected by the combination of the resistors because the resistors are in parallel and therefore have the same voltage v_o. From Fig. 2.42(b), we can obtain v_o in two ways. One way is to apply Ohm's law to get

$$i = \frac{12}{4+2} = 2 \text{ A}$$

and hence, $v_o = 2i = 2 \times 2 = 4$ V. Another way is to apply voltage division, since the 12 V in Fig. 2.42(b) is divided between the 4-Ω and 2-Ω resistors. Hence,

$$v_o = \frac{2}{2+4}(12 \text{ V}) = 4 \text{ V}$$

Similarly, i_o can be obtained in two ways. One approach is to apply Ohm's law to the 3-Ω resistor in Fig. 2.42(a) now that we know v_o; thus,

$$v_o = 3i_o = 4 \qquad \Longrightarrow \qquad i_o = \frac{4}{3} \text{ A}$$

Another approach is to apply current division to the circuit in Fig. 2.42(a) now that we know i, by writing

$$i_o = \frac{6}{6+3} i = \frac{2}{3}(2 \text{ A}) = \frac{4}{3} \text{ A}$$

The power dissipated in the 3-Ω resistor is

$$p_o = v_o i_o = 4\left(\frac{4}{3}\right) = 5.333 \text{ W}$$

PRACTICE PROBLEM 2.12

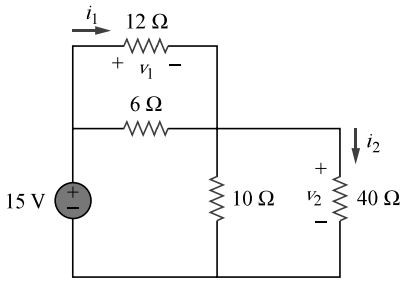


Figure 2.43 For Practice Prob. 2.12.

Find v_1 and v_2 in the circuit shown in Fig. 2.43. Also calculate i_1 and i_2 and the power dissipated in the 12-Ω and 40-Ω resistors.

Answer: $v_1 = 5$ V, $i_1 = 416.7$ mA, $p_1 = 2.083$ W, $v_2 = 10$ V, $i_2 = 250$ mA, $p_2 = 2.5$ W.

EXAMPLE 2.13

For the circuit shown in Fig. 2.44(a), determine: (a) the voltage v_o, (b) the power supplied by the current source, (c) the power absorbed by each resistor.

Solution:

(a) The 6-kΩ and 12-kΩ resistors are in series so that their combined value is $6 + 12 = 18$ kΩ. Thus the circuit in Fig. 2.44(a) reduces to that

shown in Fig. 2.44(b). We now apply the current division technique to find i_1 and i_2.

$$i_1 = \frac{18{,}000}{9000 + 18{,}000}(30\text{ mA}) = 20\text{ mA}$$

$$i_2 = \frac{9000}{9000 + 18{,}000}(30\text{ A}) = 10\text{ mA}$$

Notice that the voltage across the 9-kΩ and 18-kΩ resistors is the same, and $v_o = 9{,}000i_1 = 18{,}000i_2 = 180$ V, as expected.
(b) Power supplied by the source is

$$p_o = v_o i_o = 180(30)\text{ mW} = 5.4\text{ W}$$

(c) Power absorbed by the 12-kΩ resistor is

$$p = iv = i_2(i_2R) = i_2^2R = (10 \times 10^{-3})^2(12{,}000) = 1.2\text{ W}$$

Power absorbed by the 6-kΩ resistor is

$$p = i_2^2R = (10 \times 10^{-3})^2(6000) = 0.6\text{ W}$$

Power absorbed by the 9-kΩ resistor is

$$p = \frac{v_o^2}{R} = \frac{(180)^2}{9000} = 3.6\text{ W}$$

or

$$p = v_o i_1 = 180(20)\text{ mW} = 3.6\text{ W}$$

Notice that the power supplied (5.4 W) equals the power absorbed ($1.2 + 0.6 + 3.6 = 5.4$ W). This is one way of checking results.

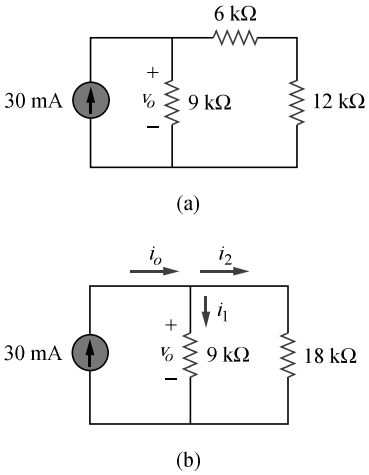


Figure 2.44 For Example 2.13: (a) original circuit, (b) its equivalent circuit.

PRACTICE PROBLEM 2.13

For the circuit shown in Fig. 2.45, find: (a) v_1 and v_2, (b) the power dissipated in the 3-kΩ and 20-kΩ resistors, and (c) the power supplied by the current source.

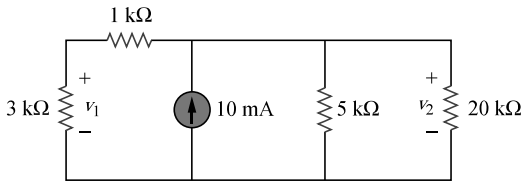


Figure 2.45 For Practice Prob. 2.13.

Answer: (a) 15 V, 20 V, (b) 75 mW, 20 mW, (c) 200 mW.

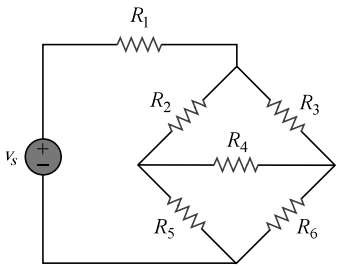


Figure 2.46 The bridge network.

†2.7 WYE-DELTA TRANSFORMATIONS

Situations often arise in circuit analysis when the resistors are neither in parallel nor in series. For example, consider the bridge circuit in Fig. 2.46. How do we combine resistors R_1 through R_6 when the resistors are neither in series nor in parallel? Many circuits of the type shown in Fig. 2.46 can be simplified by using three-terminal equivalent networks. These are the wye (Y) or tee (T) network shown in Fig. 2.47 and the delta (Δ) or pi (Π) network shown in Fig. 2.48. These networks occur by themselves or as part of a larger network. They are used in three-phase networks, electrical filters, and matching networks. Our main interest here is in how to identify them when they occur as part of a network and how to apply wye-delta transformation in the analysis of that network.

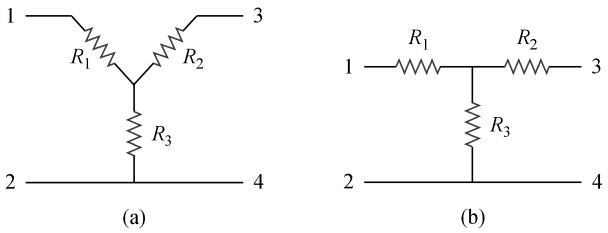


Figure 2.47 Two forms of the same network: (a) Y, (b) T.

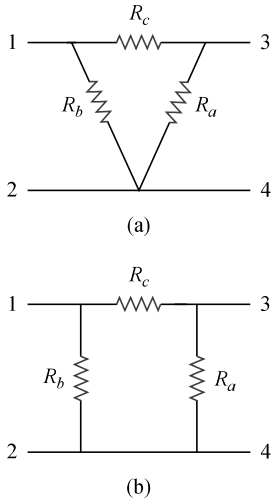


Figure 2.48 Two forms of the same network: (a) Δ, (b) Π.

Delta to Wye Conversion

Suppose it is more convenient to work with a wye network in a place where the circuit contains a delta configuration. We superimpose a wye network on the existing delta network and find the equivalent resistances in the wye network. To obtain the equivalent resistances in the wye network, we compare the two networks and make sure that the resistance between each pair of nodes in the Δ (or Π) network is the same as the resistance between the same pair of nodes in the Y (or T) network. For terminals 1 and 2 in Figs. 2.47 and 2.48, for example,

$$\begin{aligned} R_{12}(\mathrm{Y}) &= R_1 + R_3 \\ R_{12}(\Delta) &= R_b \parallel (R_a + R_c) \end{aligned} \tag{2.46}$$

Setting $R_{12}(\mathrm{Y}) = R_{12}(\Delta)$ gives

$$R_{12} = R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} \tag{2.47a}$$

Similarly,

$$R_{13} = R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} \tag{2.47b}$$

$$R_{34} = R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \tag{2.47c}$$

Subtracting Eq. (2.47c) from Eq. (2.47a), we get

$$R_1 - R_2 = \frac{R_c(R_b - R_a)}{R_a + R_b + R_c} \tag{2.48}$$

Adding Eqs. (2.47b) and (2.48) gives

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \tag{2.49}$$

and subtracting Eq. (2.48) from Eq. (2.47b) yields

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} \tag{2.50}$$

Subtracting Eq. (2.49) from Eq. (2.47a), we obtain

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} \tag{2.51}$$

We do not need to memorize Eqs. (2.49) to (2.51). To transform a Δ network to Y, we create an extra node n as shown in Fig. 2.49 and follow this conversion rule:

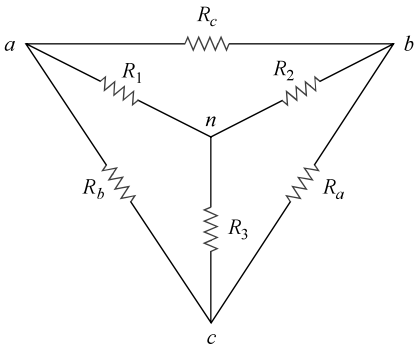


Figure 2.49 Superposition of Y and Δ networks as an aid in transforming one to the other.

> Each resistor in the Y network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three Δ resistors.

Wye to Delta Conversion

To obtain the conversion formulas for transforming a wye network to an equivalent delta network, we note from Eqs. (2.49) to (2.51) that

$$\begin{aligned} R_1 R_2 + R_2 R_3 + R_3 R_1 &= \frac{R_a R_b R_c (R_a + R_b + R_c)}{(R_a + R_b + R_c)^2} \\ &= \frac{R_a R_b R_c}{R_a + R_b + R_c} \end{aligned} \tag{2.52}$$

Dividing Eq. (2.52) by each of Eqs. (2.49) to (2.51) leads to the following equations:

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} \tag{2.53}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} \tag{2.54}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} \tag{2.55}$$

From Eqs. (2.53) to (2.55) and Fig. 2.49, the conversion rule for Y to Δ is as follows:

Each resistor in the Δ network is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor.

The Y and Δ networks are said to be *balanced* when

$$R_1 = R_2 = R_3 = R_Y, \qquad R_a = R_b = R_c = R_\Delta \tag{2.56}$$

Under these conditions, conversion formulas become

$$\boxed{R_Y = \frac{R_\Delta}{3} \quad \text{or} \quad R_\Delta = 3R_Y} \tag{2.57}$$

One may wonder why R_Y is less than R_Δ. Well, we notice that the Y-connection is like a "series" connection while the Δ-connection is like a "parallel" connection.

Note that in making the transformation, we do not take anything out of the circuit or put in anything new. We are merely substituting different but mathematically equivalent three-terminal network patterns to create a circuit in which resistors are either in series or in parallel, allowing us to calculate R_{eq} if necessary.

EXAMPLE 2.14

Convert the Δ network in Fig. 2.50(a) to an equivalent Y network.

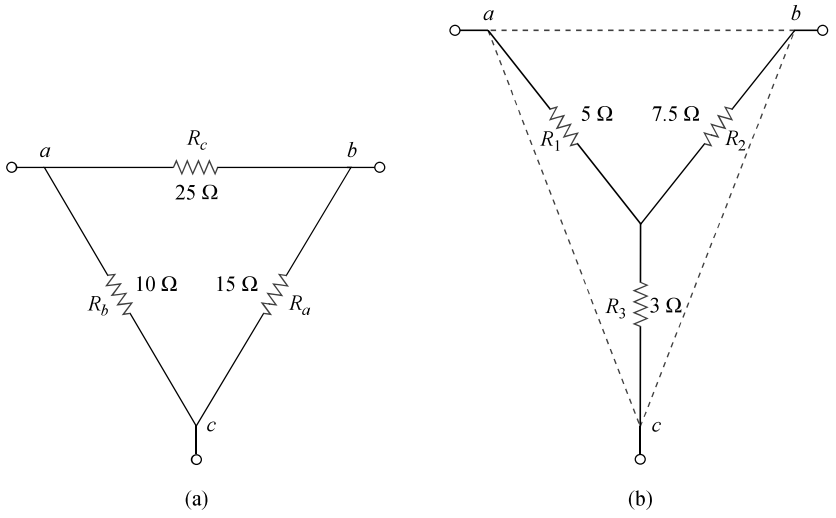


Figure 2.50 For Example 2.14: (a) original Δ network, (b) Y equivalent network.

Solution:

Using Eqs. (2.49) to (2.51), we obtain

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{25 \times 10}{25 + 10 + 15} = \frac{250}{50} = 5\ \Omega$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5\ \Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3\ \Omega$$

The equivalent Y network is shown in Fig. 2.50(b).

PRACTICE PROBLEM 2.14

Transform the wye network in Fig. 2.51 to a delta network.

Answer: $R_a = 140\ \Omega$, $R_b = 70\ \Omega$, $R_c = 35\ \Omega$.

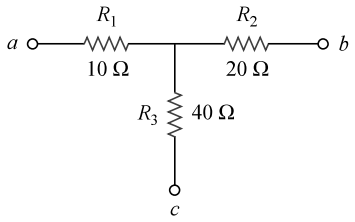


Figure 2.51 For Practice Prob. 2.14.

EXAMPLE 2.15

Obtain the equivalent resistance R_{ab} for the circuit in Fig. 2.52 and use it to find current i.

Solution:

In this circuit, there are two Y networks and one Δ network. Transforming just one of these will simplify the circuit. If we convert the Y network comprising the 5-Ω, 10-Ω, and 20-Ω resistors, we may select

$$R_1 = 10\ \Omega, \qquad R_2 = 20\ \Omega, \qquad R_3 = 5\ \Omega$$

Thus from Eqs. (2.53) to (2.55) we have

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} = \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{10}$$

$$= \frac{350}{10} = 35\ \Omega$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} = \frac{350}{20} = 17.5\ \Omega$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} = \frac{350}{5} = 70\ \Omega$$

With the Y converted to Δ, the equivalent circuit (with the voltage source removed for now) is shown in Fig. 2.53(a). Combining the three pairs of resistors in parallel, we obtain

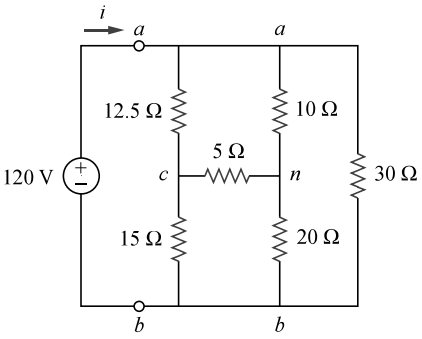


Figure 2.52 For Example 2.15.

$$70 \parallel 30 = \frac{70 \times 30}{70 + 30} = 21\ \Omega$$

$$12.5 \parallel 17.5 = \frac{12.5 \times 17.5}{12.5 + 17.5} = 7.2917\ \Omega$$

$$15 \parallel 35 = \frac{15 \times 35}{15 + 35} = 10.5\ \Omega$$

so that the equivalent circuit is shown in Fig. 2.53(b). Hence, we find

$$R_{ab} = (7.292 + 10.5) \parallel 21 = \frac{17.792 \times 21}{17.792 + 21} = 9.632\ \Omega$$

Then

$$i = \frac{v_s}{R_{ab}} = \frac{120}{9.632} = 12.458\ \text{A}$$

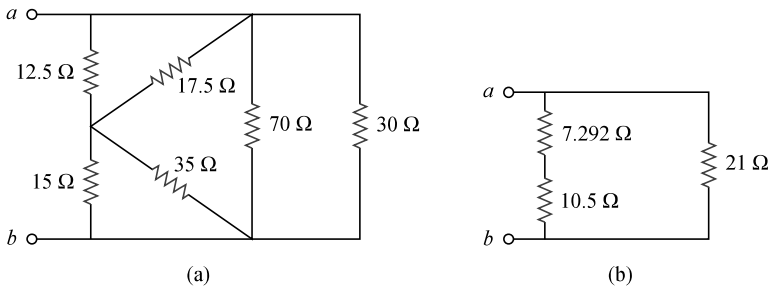


Figure 2.53 Equivalent circuits to Fig. 2.52, with the voltage removed.

PRACTICE PROBLEM 2.15

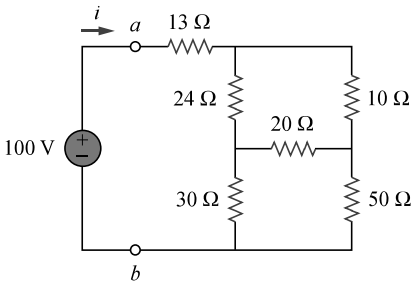


Figure 2.54 For Practice Prob. 2.15.

For the bridge network in Fig. 2.54, find R_{ab} and i.

Answer: 40 Ω, 2.5 A.

†2.8 APPLICATIONS

Resistors are often used to model devices that convert electrical energy into heat or other forms of energy. Such devices include conducting wire, lightbulbs, electric heaters, stoves, ovens, and loudspeakers. In this

section, we will consider two real-life problems that apply the concepts developed in this chapter: electrical lighting systems and design of dc meters.

So far, we have assumed that connecting wires are perfect conductors (i.e., conductors of zero resistance). In real physical systems, however, the resistance of the connecting wire may be appreciably large, and the modeling of the system must include that resistance.

2.8.1 Lighting Systems

Lighting systems, such as in a house or on a Christmas tree, often consist of N lamps connected either in parallel or in series, as shown in Fig. 2.55. Each lamp is modeled as a resistor. Assuming that all the lamps are identical and V_o is the power-line voltage, the voltage across each lamp is V_o for the parallel connection and V_o/N for the series connection. The series connection is easy to manufacture but is seldom used in practice, for at least two reasons. First, it is less reliable; when a lamp fails, all the lamps go out. Second, it is harder to maintain; when a lamp is bad, one must test all the lamps one by one to detect the faulty one.

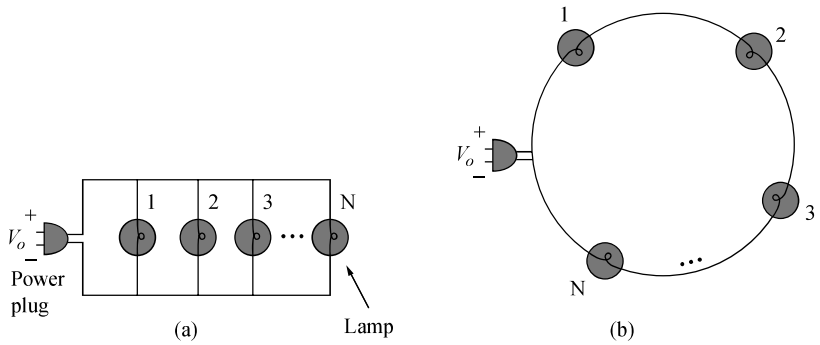


Figure 2.55 (a) Parallel connection of lightbulbs, (b) series connection of lightbulbs.

EXAMPLE 2.16

Three lightbulbs are connected to a 9-V battery as shown in Fig. 2.56(a). Calculate: (a) the total current supplied by the battery, (b) the current through each bulb, (c) the resistance of each bulb.

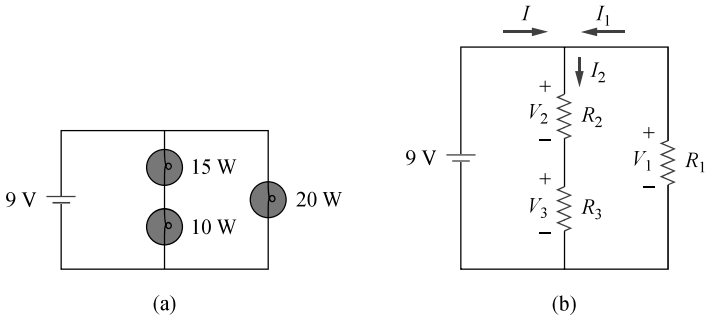


Figure 2.56 (a) Lighting system with three bulbs, (b) resistive circuit equivalent model.

Solution:

(a) The total power supplied by the battery is equal to the total power absorbed by the bulbs, that is,

$$p = 15 + 10 + 20 = 45 \text{ W}$$

Since $p = VI$, then the total current supplied by the battery is

$$I = \frac{p}{V} = \frac{45}{9} = 5 \text{ A}$$

(b) The bulbs can be modeled as resistors as shown in Fig. 2.56(b). Since R_1 (20-W bulb) is in parallel with the battery as well as the series combination of R_2 and R_3,

$$V_1 = V_2 + V_3 = 9 \text{ V}$$

The current through R_1 is

$$I_1 = \frac{p_1}{V_1} = \frac{20}{9} = 2.222 \text{ A}$$

By KCL, the current through the series combination of R_2 and R_3 is

$$I_2 = I - I_1 = 5 - 2.222 = 2.778 \text{ A}$$

(c) Since $p = I^2R$,

$$R_1 = \frac{p_1}{I_1^2} = \frac{20}{2.222^2} = 4.05 \ \Omega$$

$$R_2 = \frac{p_2}{I_2^2} = \frac{15}{2.777^2} = 1.945 \ \Omega$$

$$R_3 = \frac{p_3}{I_3^2} = \frac{10}{2.777^2} = 1.297 \ \Omega$$

PRACTICE PROBLEM 2.16

Refer to Fig. 2.55 and assume there are 10 lightbulbs, each with a power rating of 40 W. If the voltage at the plug is 110 V for the parallel and series connections, calculate the current through each bulb for both cases.

Answer: 0.364 A (parallel), 3.64 A (series).

2.8.2 Design of DC Meters

By their nature, resistors are used to control the flow of current. We take advantage of this property in several applications, such as in a potentiometer (Fig. 2.57). The word *potentiometer*, derived from the words *potential* and *meter*, implies that potential can be metered out. The potentiometer (or pot for short) is a three-terminal device that operates on the principle of voltage division. It is essentially an adjustable voltage divider. As a voltage regulator, it is used as a volume or level control on radios, TVs, and other devices. In Fig. 2.57,

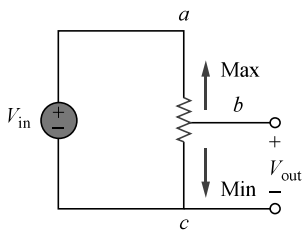


Figure 2.57 The potentiometer controlling potential levels.

$$V_{out} = V_{bc} = \frac{R_{bc}}{R_{ac}} V_{in} \tag{2.58}$$

where $R_{ac} = R_{ab} + R_{bc}$. Thus, V_{out} decreases or increases as the sliding contact of the pot moves toward c or a, respectively.

Another application where resistors are used to control current flow is in the analog dc meters—the ammeter, voltmeter, and ohmmeter, which measure current, voltage, and resistance, respectively. Each of these meters employs the d'Arsonval meter movement, shown in Fig. 2.58. The movement consists essentially of a movable iron-core coil mounted on a pivot between the poles of a permanent magnet. When current flows through the coil, it creates a torque which causes the pointer to deflect. The amount of current through the coil determines the deflection of the pointer, which is registered on a scale attached to the meter movement. For example, if the meter movement is rated 1 mA, 50 Ω, it would take 1 mA to cause a full-scale deflection of the meter movement. By introducing additional circuitry to the d'Arsonval meter movement, an ammeter, voltmeter, or ohmmeter can be constructed.

An instrument capable of measuring voltage, current, and resistance is called a *multimeter* or a *volt-ohm meter* (VOM).

Consider Fig. 2.59, where an analog voltmeter and ammeter are connected to an element. The voltmeter measures the voltage across a *load* and is therefore connected in parallel with the element. As shown in Fig. 2.60(a), the voltmeter consists of a d'Arsonval movement in parallel with a resistor whose resistance R_m is deliberately made very large (theoretically, infinite), to minimize the current drawn from the circuit. To extend the range of voltage that the meter can measure, series multiplier resistors are often connected with the voltmeters, as shown in Fig. 2.60(b). The multiple-range voltmeter in Fig. 2.60(b) can measure voltage from 0 to 1 V, 0 to 10 V, or 0 to 100 V, depending on whether the switch is connected to R_1, R_2, or R_3, respectively.

A load is a component that is receiving energy (an energy sink), as opposed to a generator supplying energy (an energy source). More about loading will be discussed in Section 4.9.1.

Let us calculate the multiplier resistor R_n for the single-range voltmeter in Fig. 2.60(a), or $R_n = R_1$, R_2, or R_3 for the multiple-range voltmeter in Fig. 2.60(b). We need to determine the value of R_n to be connected in series with the internal resistance R_m of the voltmeter. In any design, we consider the worst-case condition. In this case, the worst case occurs when the full-scale current $I_{fs} = I_m$ flows through the meter. This should also correspond to the maximum voltage reading or the full-scale voltage V_{fs}. Since the multiplier resistance R_n is in series with the

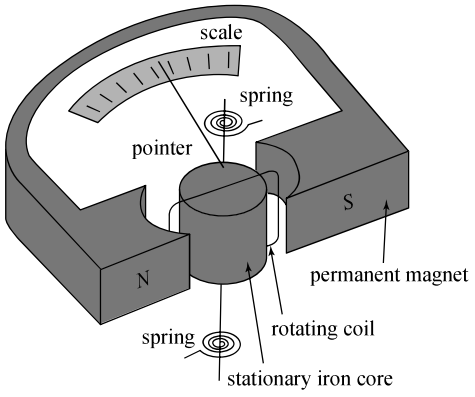


Figure 2.58 A d'Arsonval meter movement.

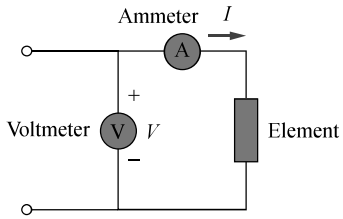


Figure 2.59 Connection of a voltmeter and an ammeter to an element.

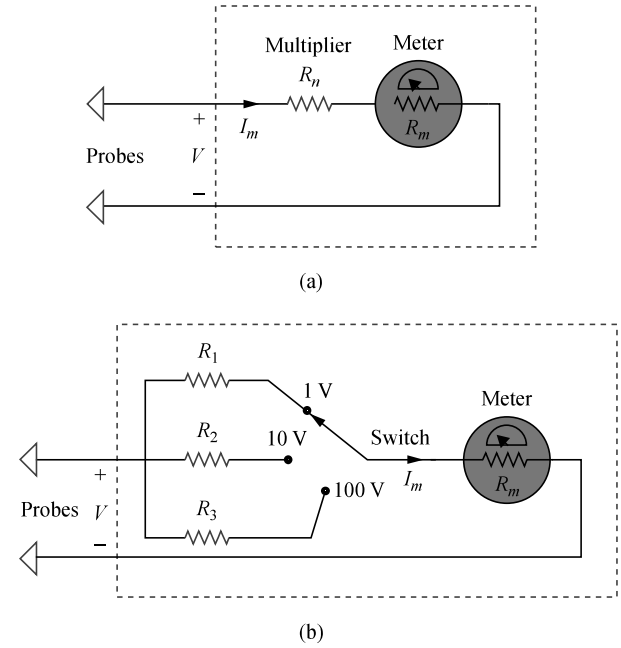


Figure 2.60 Voltmeters: (a) single-range type, (b) multiple-range type.

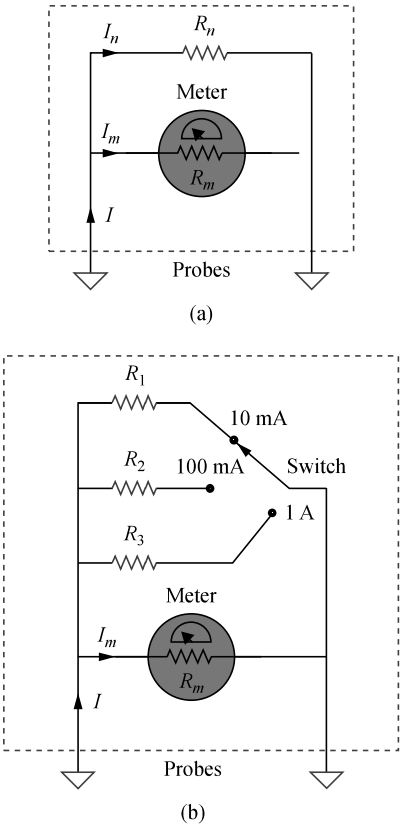


Figure 2.61 Ammeters: (a) single-range type, (b) multiple-range type.

internal resistance R_m,

$$V_{\text{fs}} = I_{\text{fs}}(R_n + R_m) \tag{2.59}$$

From this, we obtain

$$R_n = \frac{V_{\text{fs}}}{I_{\text{fs}}} - R_m \tag{2.60}$$

Similarly, the ammeter measures the current through the load and is connected in series with it. As shown in Fig. 2.61(a), the ammeter consists of a d'Arsonval movement in parallel with a resistor whose resistance R_m is deliberately made very small (theoretically, zero) to minimize the voltage drop across it. To allow multiple range, shunt resistors are often connected in parallel with R_m as shown in Fig. 2.61(b). The shunt resistors allow the meter to measure in the range 0–10 mA, 0–100 mA, or 0–1 A, depending on whether the switch is connected to R_1, R_2, or R_3, respectively.

Now our objective is to obtain the multiplier shunt R_n for the single-range ammeter in Fig. 2.61(a), or $R_n = R_1$, R_2, or R_3 for the multiple-range ammeter in Fig. 2.61(b). We notice that R_m and R_n are in parallel and that at full-scale reading $I = I_{\text{fs}} = I_m + I_n$, where I_n is the current through the shunt resistor R_n. Applying the current division principle yields

$$I_m = \frac{R_n}{R_n + R_m} I_{\text{fs}}$$

or

$$R_n = \frac{I_m}{I_{\text{fs}} - I_m} R_m \tag{2.61}$$

The resistance R_x of a linear resistor can be measured in two ways. An indirect way is to measure the current I that flows through it by connecting an ammeter in series with it and the voltage V across it by connecting a voltmeter in parallel with it, as shown in Fig. 2.62(a). Then

$$R_x = \frac{V}{I} \tag{2.62}$$

The direct method of measuring resistance is to use an ohmmeter. An ohmmeter consists basically of a d'Arsonval movement, a variable resistor or potentiometer, and a battery, as shown in Fig. 2.62(b). Applying KVL to the circuit in Fig. 2.62(b) gives

$$E = (R + R_m + R_x)I_m$$

or

$$R_x = \frac{E}{I_m} - (R + R_m) \tag{2.63}$$

The resistor R is selected such that the meter gives a full-scale deflection, that is, $I_m = I_{\text{fs}}$ when $R_x = 0$. This implies that

$$E = (R + R_m)I_{\text{fs}} \tag{2.64}$$

Substituting Eq. (2.64) into Eq. (2.63) leads to

$$R_x = \left(\frac{I_{\text{fs}}}{I_m} - 1\right)(R + R_m) \tag{2.65}$$

As mentioned, the types of meters we have discussed are known as *analog* meters and are based on the d'Arsonval meter movement. Another type of meter, called a *digital meter*, is based on active circuit elements such as op amps. For example, a digital multimeter displays measurements of dc or ac voltage, current, and resistance as discrete numbers, instead of using a pointer deflection on a continuous scale as in an analog multimeter. Digital meters are what you would most likely use in a modern lab. However, the design of digital meters is beyond the scope of this book.

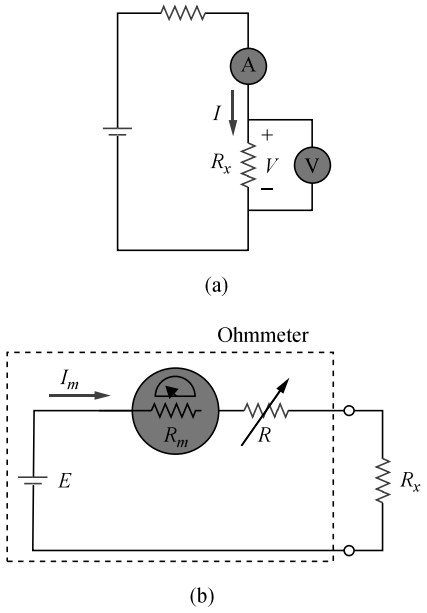


Figure 2.62 Two ways of measuring resistance: (a) using an ammeter and a voltmeter, (b) using an ohmmeter.

EXAMPLE 2.17

Following the voltmeter setup of Fig. 2.60, design a voltmeter for the following multiple ranges:
(a) 0–1 V (b) 0–5 V (c) 0–50 V (d) 0–100 V
Assume that the internal resistance $R_m = 2\ \text{k}\Omega$ and the full-scale current $I_{\text{fs}} = 100\ \mu\text{A}$.

Solution:

We apply Eq. (2.60) and assume that R_1, R_2, R_3, and R_4 correspond with ranges 0–1 V, 0–5 V, 0–50 V, and 0–100 V, respectively.
(a) For range 0–1 V,

$$R_1 = \frac{1}{100 \times 10^{-6}} - 2000 = 10{,}000 - 2000 = 8\ \text{k}\Omega$$

(b) For range 0–5 V,

$$R_2 = \frac{5}{100 \times 10^{-6}} - 2000 = 50{,}000 - 2000 = 48\ \text{k}\Omega$$

(c) For range 0–50 V,

$$R_3 = \frac{50}{100 \times 10^{-6}} - 2000 = 500{,}000 - 2000 = 498\ \text{k}\Omega$$

(d) For range 0–100 V,

$$R_4 = \frac{100\ \text{V}}{100 \times 10^{-6}} - 2000 = 1{,}000{,}000 - 2000 = 998\ \text{k}\Omega$$

Note that the ratio of the total resistance $(R_n + R_m)$ to the full-scale voltage V_{fs} is constant and equal to $1/I_{\text{fs}}$ for the four ranges. This ratio (given in ohms per volt, or Ω/V) is known as the *sensitivity* of the voltmeter. The larger the sensitivity, the better the voltmeter.

PRACTICE PROBLEM 2.17

Following the ammeter setup of Fig. 2.61, design an ammeter for the following multiple ranges:
(a) 0–1 A (b) 0–100 mA (c) 0–10 mA
Take the full-scale meter current as $I_m = 1$ mA and the internal resistance of the ammeter as $R_m = 50\ \Omega$.

Answer: Shunt resistors: 0.05 Ω, 0.505 Ω, 5.556 Ω.

2.9 SUMMARY

1. A resistor is a passive element in which the voltage v across it is directly proportional to the current i through it. That is, a resistor is a device that obeys Ohm's law,

$$v = iR$$

where R is the resistance of the resistor.

2. A short circuit is a resistor (a perfectly conducting wire) with zero resistance ($R = 0$). An open circuit is a resistor with infinite resistance ($R = \infty$).
3. The conductance G of a resistor is the reciprocal of its resistance:

$$G = \frac{1}{R}$$

4. A branch is a single two-terminal element in an electric circuit. A node is the point of connection between two or more branches. A loop is a closed path in a circuit. The number of branches b, the number of nodes n, and the number of independent loops l in a network are related as

$$b = l + n - 1$$

5. Kirchhoff's current law (KCL) states that the currents at any node algebraically sum to zero. In other words, the sum of the currents entering a node equals the sum of currents leaving the node.
6. Kirchhoff's voltage law (KVL) states that the voltages around a closed path algebraically sum to zero. In other words, the sum of voltage rises equals the sum of voltage drops.
7. Two elements are in series when they are connected sequentially, end to end. When elements are in series, the same current flows through them ($i_1 = i_2$). They are in parallel if they are connected to the same two nodes. Elements in parallel always have the same voltage across them ($v_1 = v_2$).
8. When two resistors R_1 ($= 1/G_1$) and R_2 ($= 1/G_2$) are in series, their equivalent resistance R_{eq} and equivalent conductance G_{eq} are

$$R_{eq} = R_1 + R_2, \qquad G_{eq} = \frac{G_1 G_2}{G_1 + G_2}$$

9. When two resistors R_1 ($= 1/G_1$) and R_2 ($= 1/G_2$) are in parallel, their equivalent resistance R_{eq} and equivalent conductance G_{eq} are

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}, \qquad G_{eq} = G_1 + G_2$$

10. The voltage division principle for two resistors in series is

$$v_1 = \frac{R_1}{R_1 + R_2} v, \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$

11. The current division principle for two resistors in parallel is

$$i_1 = \frac{R_2}{R_1 + R_2} i, \qquad i_2 = \frac{R_1}{R_1 + R_2} i$$

12. The formulas for a delta-to-wye transformation are

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}, \qquad R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

13. The formulas for a wye-to-delta transformation are

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}, \qquad R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

14. The basic laws covered in this chapter can be applied to the problems of electrical lighting and design of dc meters.

REVIEW QUESTIONS

2.1 The reciprocal of resistance is:
(a) voltage (b) current
(c) conductance (d) coulombs

2.2 An electric heater draws 10 A from a 120-V line. The resistance of the heater is:
(a) 1200 Ω (b) 120 Ω
(c) 12 Ω (d) 1.2 Ω

2.3 The voltage drop across a 1.5-kW toaster that draws 12 A of current is:
(a) 18 kV (b) 125 V
(c) 120 V (d) 10.42 V

2.4 The maximum current that a 2W, 80 kΩ resistor can safely conduct is:
(a) 160 kA (b) 40 kA
(c) 5 mA (d) 25 μA

2.5 A network has 12 branches and 8 independent loops. How many nodes are there in the network?
(a) 19 (b) 17 (c) 5 (d) 4

2.6 The current I in the circuit in Fig. 2.63 is:
(a) −0.8 A (b) −0.2 A
(c) 0.2 A (d) 0.8 A

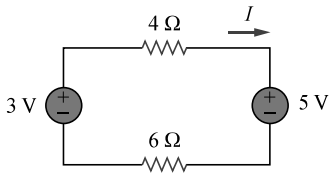


Figure 2.63 For Review Question 2.6.

2.7 The current I_o in Fig. 2.64 is:
(a) −4 A (b) −2 A (c) 4 A (d) 16 A

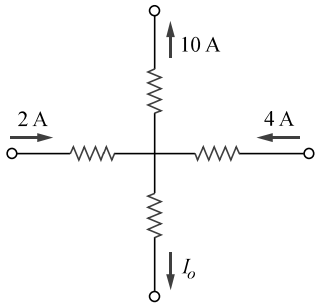


Figure 2.64 For Review Question 2.7.

2.8 In the circuit in Fig. 2.65, V is:
(a) 30 V (b) 14 V (c) 10 V (d) 6 V

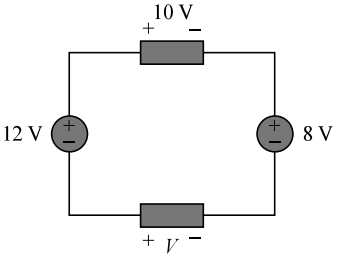


Figure 2.65 For Review Question 2.8.

2.9 Which of the circuits in Fig. 2.66 will give you $V_{ab} = 7$ V?

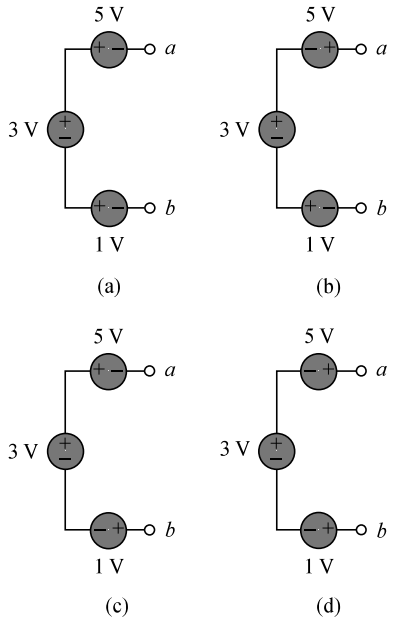


Figure 2.66 For Review Question 2.9.

2.10 The equivalent resistance of the circuit in Fig. 2.67 is:
(a) 4 kΩ (b) 5 kΩ (c) 8 kΩ (d) 14 kΩ

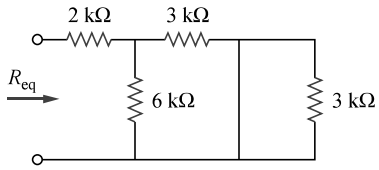


Figure 2.67 For Review Question 2.10.

Answers: 2.1c, 2.2c, 2.3b, 2.4c, 2.5c, 2.6b, 2.7a, 2.8d, 2.9d, 2.10a.

PROBLEMS

Section 2.2 Ohm's Law

2.1 The voltage across a 5-kΩ resistor is 16 V. Find the current through the resistor.

2.2 Find the hot resistance of a lightbulb rated 60 W, 120 V.

2.3 When the voltage across a resistor is 120 V, the current through it is 2.5 mA. Calculate its conductance.

2.4 (a) Calculate current i in Fig. 2.68 when the switch is in position 1.
(b) Find the current when the switch is in position 2.

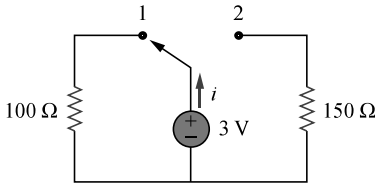


Figure 2.68 For Prob. 2.4.

Section 2.3 Nodes, Branches, and Loops

2.5 For the network graph in Fig. 2.69, find the number of nodes, branches, and loops.

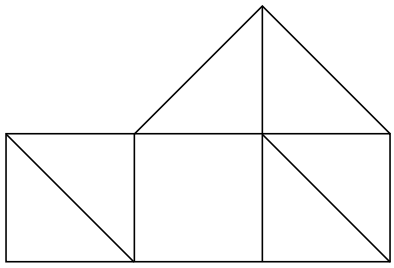


Figure 2.69 For Prob. 2.5.

2.6 In the network graph shown in Fig. 2.70, determine the number of branches and nodes.

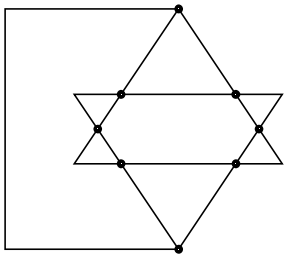


Figure 2.70 For Prob. 2.6.

2.7 Determine the number of branches and nodes in the circuit in Fig. 2.71.

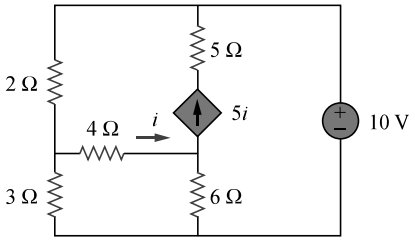


Figure 2.71 For Prob. 2.7.

Section 2.4 Kirchhoff's Laws

2.8 Use KCL to obtain currents i_1, i_2, and i_3 in the circuit shown in Fig. 2.72.

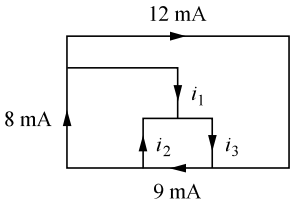


Figure 2.72 For Prob. 2.8.

2.9 Find i_1, i_2, and i_3 in the circuit in Fig. 2.73.

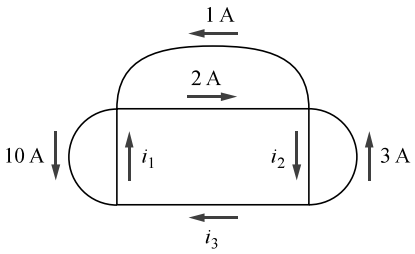


Figure 2.73 For Prob. 2.9.

2.10 Determine i_1 and i_2 in the circuit in Fig. 2.74.

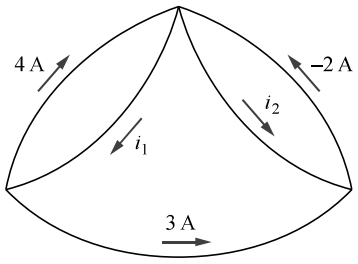


Figure 2.74 For Prob. 2.10.

2.11 Determine v_1 through v_4 in the circuit in Fig. 2.75.

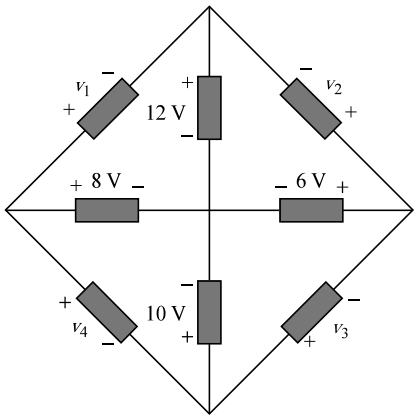


Figure 2.75 For Prob. 2.11.

2.12 In the circuit in Fig. 2.76, obtain v_1, v_2, and v_3.

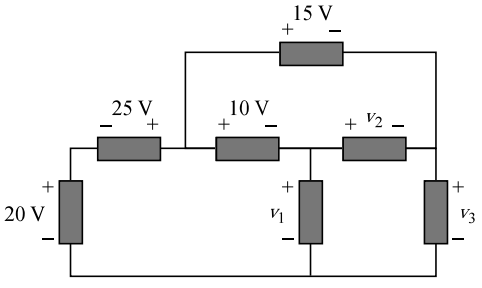


Figure 2.76 For Prob. 2.12.

2.13 Find v_1 and v_2 in the circuit in Fig. 2.77.

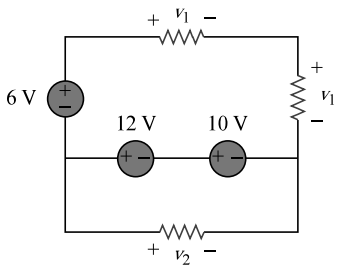


Figure 2.77 For Prob. 2.13.

2.14 Obtain v_1 through v_3 in the circuit of Fig. 2.78.

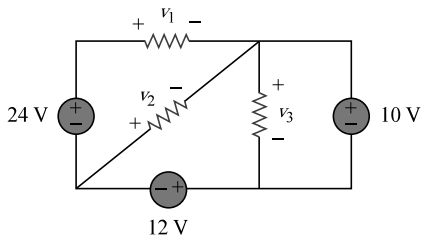


Figure 2.78 For Prob. 2.14.

2.15 Find I and V_{ab} in the circuit of Fig. 2.79.

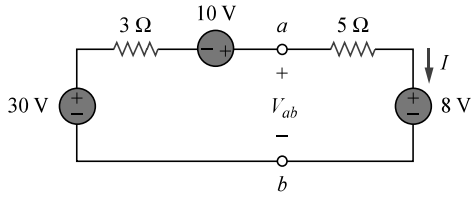


Figure 2.79 For Prob. 2.15.

2.16 From the circuit in Fig. 2.80, find I, the power dissipated by the resistor, and the power supplied by each source.

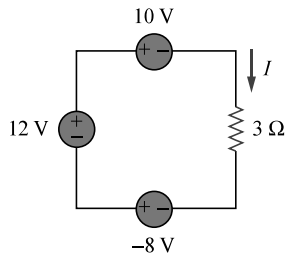


Figure 2.80 For Prob. 2.16.

2.17 Determine i_o in the circuit of Fig. 2.81.

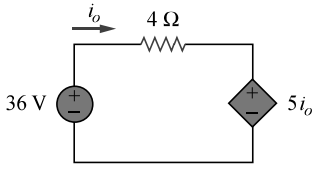


Figure 2.81 For Prob. 2.17.

2.18 Calculate the power dissipated in the 5-Ω resistor in the circuit of Fig. 2.82.

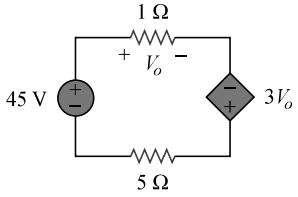


Figure 2.82 For Prob. 2.18.

2.19 Find V_o in the circuit in Fig. 2.83 and the power dissipated by the controlled source.

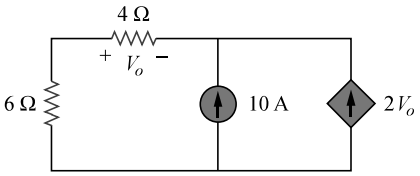


Figure 2.83 For Prob. 2.19.

2.20 For the circuit in Fig. 2.84, find V_o/V_s in terms of α, R_1, R_2, R_3, and R_4. If $R_1 = R_2 = R_3 = R_4$, what value of α will produce $|V_o/V_s| = 10$?

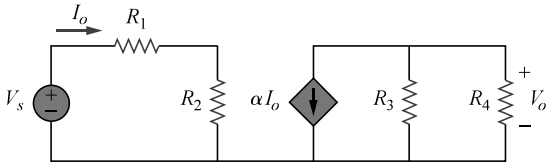


Figure 2.84 For Prob. 2.20.

2.21 For the network in Fig. 2.85, find the current, voltage, and power associated with the 20-kΩ resistor.

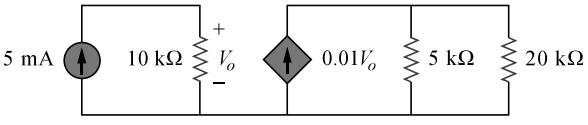


Figure 2.85 For Prob. 2.21.

Sections 2.5 and 2.6 Series and Parallel Resistors

2.22 For the circuit in Fig. 2.86, find i_1 and i_2.

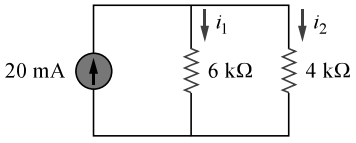


Figure 2.86 For Prob. 2.22.

2.23 Find v_1 and v_2 in the circuit in Fig. 2.87.

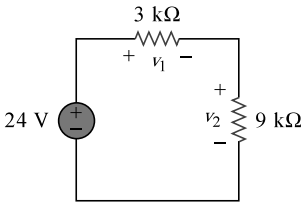


Figure 2.87 For Prob. 2.23.

2.24 Find v_1, v_2, and v_3 in the circuit in Fig. 2.88.

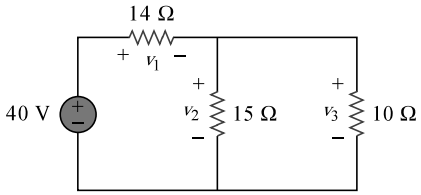


Figure 2.88 For Prob. 2.24.

2.25 Calculate v_1, i_1, v_2, and i_2 in the circuit of Fig. 2.89.

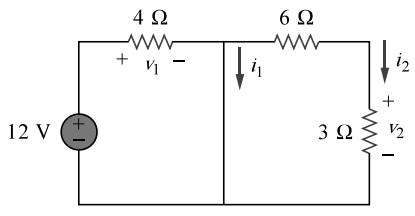


Figure 2.89 For Prob. 2.25.

2.26 Find i, v, and the power dissipated in the 6-Ω resistor in Fig. 2.90.

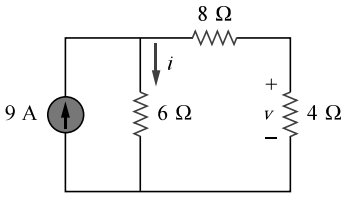


Figure 2.90 For Prob. 2.26.

2.27 In the circuit in Fig. 2.91, find v, i, and the power absorbed by the 4-Ω resistor.

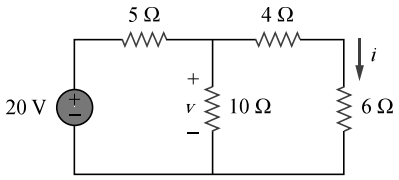


Figure 2.91 For Prob. 2.27.

2.28 Find i_1 through i_4 in the circuit in Fig. 2.92.

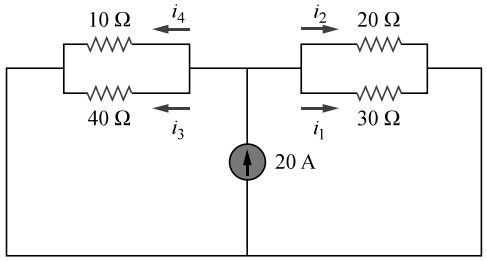


Figure 2.92 For Prob. 2.28.

2.29 Obtain v and i in the circuit in Fig. 2.93.

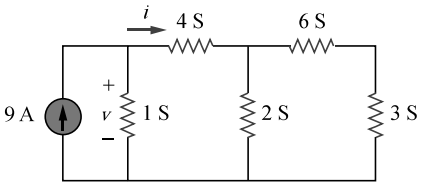


Figure 2.93 For Prob. 2.29.

2.30 Determine i_1, i_2, v_1, and v_2 in the ladder network in Fig. 2.94. Calculate the power dissipated in the 2-Ω resistor.

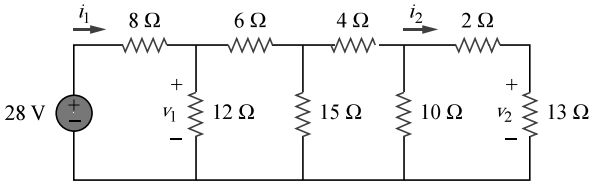


Figure 2.94 For Prob. 2.30.

2.31 Calculate V_o and I_o in the circuit of Fig. 2.95.

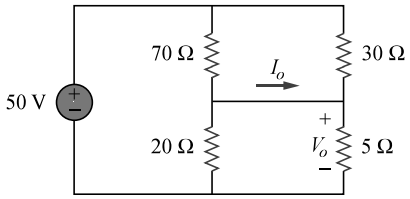


Figure 2.95 For Prob. 2.31.

2.32 Find V_o and I_o in the circuit of Fig. 2.96.

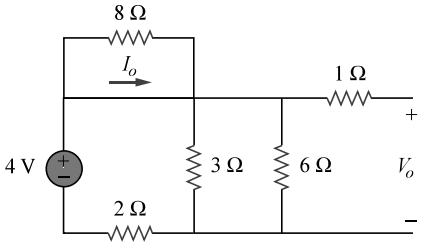


Figure 2.96 For Prob. 2.32.

2.33 In the circuit of Fig. 2.97, find R if $V_o = 4V$.

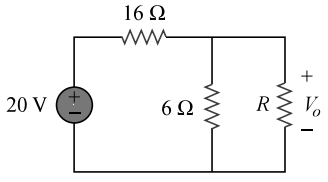


Figure 2.97 For Prob. 2.33.

2.34 Find I and V_s in the circuit of Fig. 2.98 if the current through the 3-Ω resistor is 2 A.

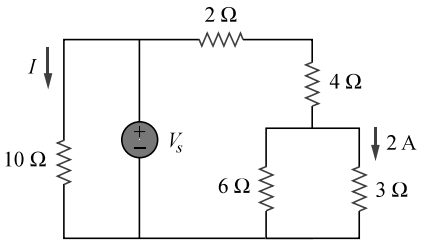


Figure 2.98 For Prob. 2.34.

2.35 Find the equivalent resistance at terminals a-b for each of the networks in Fig. 2.99.

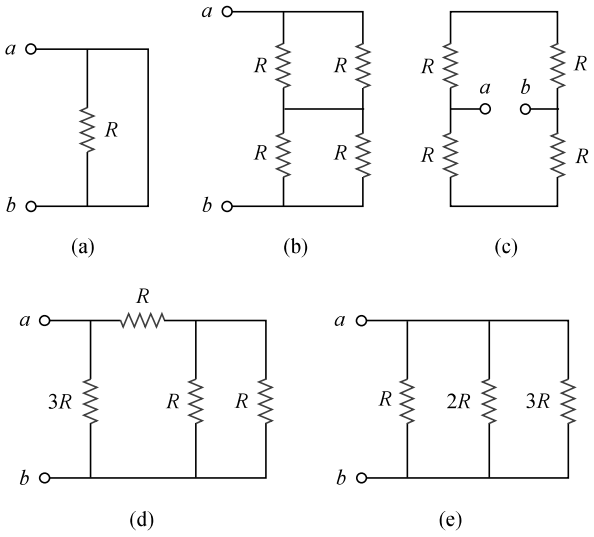


Figure 2.99 For Prob. 2.35.

2.36 For the ladder network in Fig. 2.100, find I and R_{eq}.

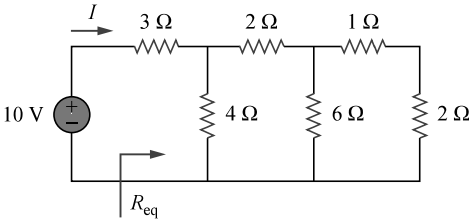


Figure 2.100 For Prob. 2.36.

2.37 If $R_{eq} = 50\ \Omega$ in the circuit in Fig. 2.101, find R.

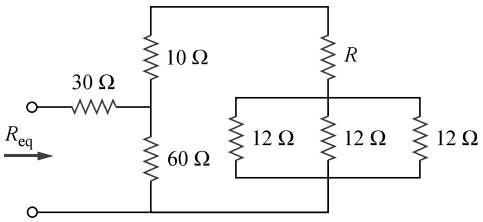


Figure 2.101 For Prob. 2.37.

2.38 Reduce each of the circuits in Fig. 2.102 to a single resistor at terminals a-b.

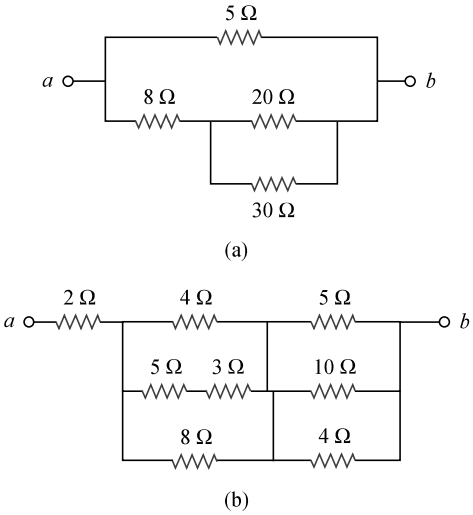


Figure 2.102 For Prob. 2.38.

2.39 Calculate the equivalent resistance R_{ab} at terminals a-b for each of the circuits in Fig. 2.103.

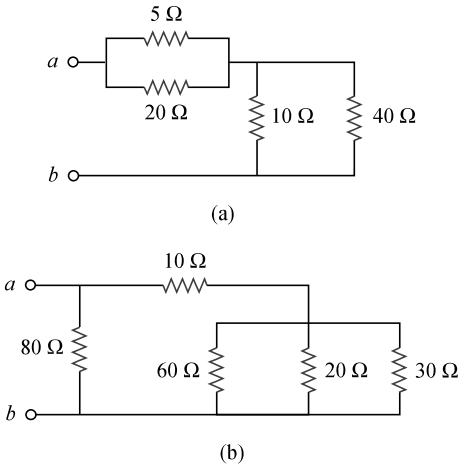


Figure 2.103 For Prob. 2.39.

2.40 Obtain the equivalent resistance at the terminals *a*-*b* for each of the circuits in Fig. 2.104.

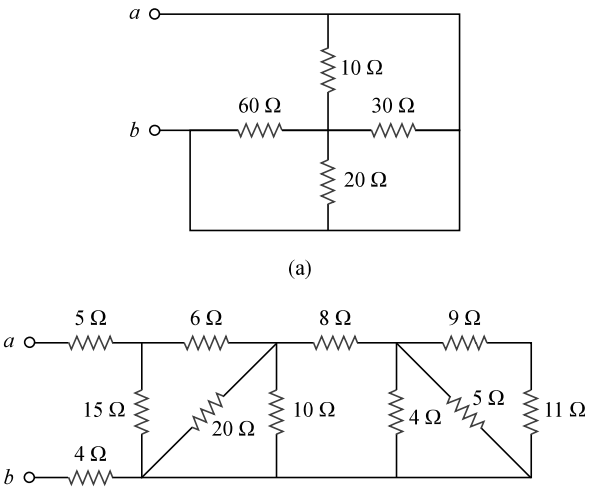


Figure 2.104 For Prob. 2.40.

2.41 Find R_{eq} at terminals *a*-*b* for each of the circuits in Fig. 2.105.

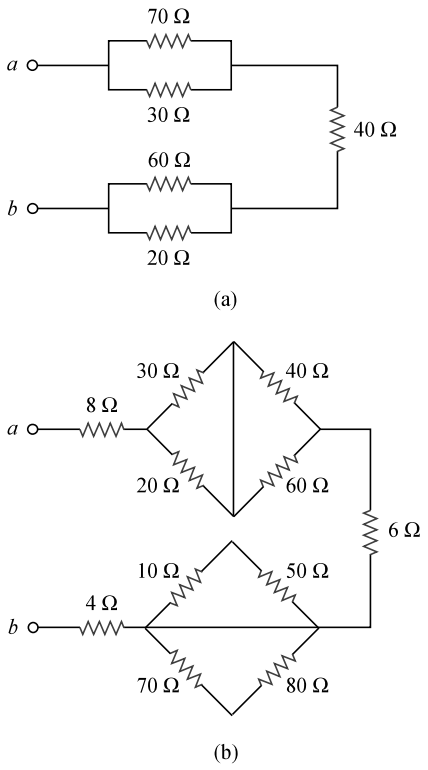


Figure 2.105 For Prob. 2.41.

2.42 Find the equivalent resistance R_{ab} in the circuit of Fig. 2.106.

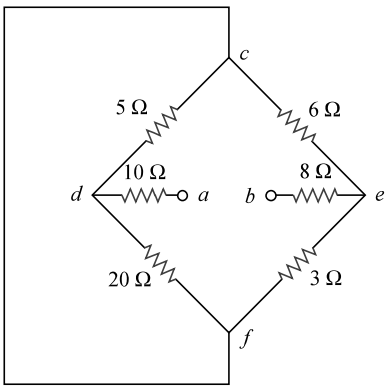


Figure 2.106 For Prob. 2.42.

Section 2.7 Wye-Delta Transformations

2.43 Convert the circuits in Fig. 2.107 from Y to Δ.

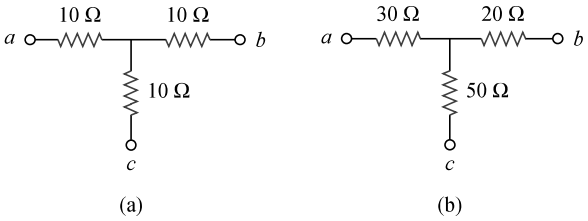


Figure 2.107 For Prob. 2.43.

2.44 Transform the circuits in Fig. 2.108 from Δ to Y.

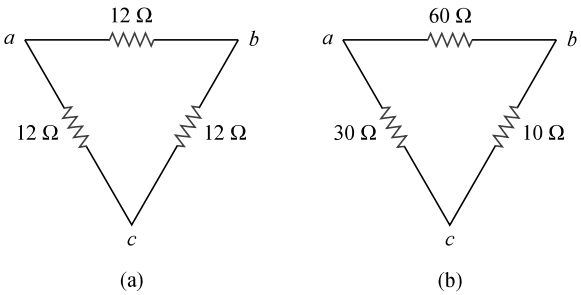


Figure 2.108 For Prob. 2.44.

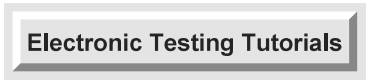
3.1 INTRODUCTION

Having understood the fundamental laws of circuit theory (Ohm's law and Kirchhoff's laws), we are now prepared to apply these laws to develop two powerful techniques for circuit analysis: nodal analysis, which is based on a systematic application of Kirchhoff's current law (KCL), and mesh analysis, which is based on a systematic application of Kirchhoff's voltage law (KVL). The two techniques are so important that this chapter should be regarded as the most important in the book. Students are therefore encouraged to pay careful attention.

With the two techniques to be developed in this chapter, we can analyze almost any circuit by obtaining a set of simultaneous equations that are then solved to obtain the required values of current or voltage. One method of solving simultaneous equations involves Cramer's rule, which allows us to calculate circuit variables as a quotient of determinants. The examples in the chapter will illustrate this method; Appendix A also briefly summarizes the essentials the reader needs to know for applying Cramer's rule.

Also in this chapter, we introduce the use of *PSpice for Windows*, a circuit simulation computer software program that we will use throughout the text. Finally, we apply the techniques learned in this chapter to analyze transistor circuits.





3.2 NODAL ANALYSIS

Nodal analysis is also known as the *node-voltage method.*

Nodal analysis provides a general procedure for analyzing circuits using node voltages as the circuit variables. Choosing node voltages instead of element voltages as circuit variables is convenient and reduces the number of equations one must solve simultaneously.

To simplify matters, we shall assume in this section that circuits do not contain voltage sources. Circuits that contain voltage sources will be analyzed in the next section.

In *nodal analysis*, we are interested in finding the node voltages. Given a circuit with n nodes without voltage sources, the nodal analysis of the circuit involves taking the following three steps.

Steps to Determine Node Voltages:

1. Select a node as the reference node. Assign voltages $v_1, v_2, \ldots, v_{n-1}$ to the remaining $n - 1$ nodes. The voltages are referenced with respect to the reference node.
2. Apply KCL to each of the $n - 1$ nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

We shall now explain and apply these three steps.

The first step in nodal analysis is selecting a node as the *reference* or *datum node*. The reference node is commonly called the *ground* since it is assumed to have zero potential. A reference node is indicated by

any of the three symbols in Fig. 3.1. The type of ground in Fig. 3.1(b) is called a *chassis ground* and is used in devices where the case, enclosure, or chassis acts as a reference point for all circuits. When the potential of the earth is used as reference, we use the *earth ground* in Fig. 3.1(a) or (c). We shall always use the symbol in Fig. 3.1(b).

Once we have selected a reference node, we assign voltage designations to nonreference nodes. Consider, for example, the circuit in Fig. 3.2(a). Node 0 is the reference node ($v = 0$), while nodes 1 and 2 are assigned voltages v_1 and v_2, respectively. Keep in mind that the node voltages are defined with respect to the reference node. As illustrated in Fig. 3.2(a), each node voltage is the voltage rise from the reference node to the corresponding nonreference node or simply the voltage of that node with respect to the reference node.

As the second step, we apply KCL to each nonreference node in the circuit. To avoid putting too much information on the same circuit, the circuit in Fig. 3.2(a) is redrawn in Fig. 3.2(b), where we now add i_1, i_2, and i_3 as the currents through resistors R_1, R_2, and R_3, respectively. At node 1, applying KCL gives

$$I_1 = I_2 + i_1 + i_2 \tag{3.1}$$

At node 2,

$$I_2 + i_2 = i_3 \tag{3.2}$$

We now apply Ohm's law to express the unknown currents i_1, i_2, and i_3 in terms of node voltages. The key idea to bear in mind is that, since resistance is a passive element, by the passive sign convention, current must always flow from a higher potential to a lower potential.

> Current flows from a higher potential to a lower potential in a resistor.

We can express this principle as

$$i = \frac{v_{\text{higher}} - v_{\text{lower}}}{R} \tag{3.3}$$

Note that this principle is in agreement with the way we defined resistance in Chapter 2 (see Fig. 2.1). With this in mind, we obtain from Fig. 3.2(b),

$$\begin{aligned} i_1 &= \frac{v_1 - 0}{R_1} \quad \text{or} \quad i_1 = G_1 v_1 \\ i_2 &= \frac{v_1 - v_2}{R_2} \quad \text{or} \quad i_2 = G_2(v_1 - v_2) \\ i_3 &= \frac{v_2 - 0}{R_3} \quad \text{or} \quad i_3 = G_3 v_2 \end{aligned} \tag{3.4}$$

Substituting Eq. (3.4) in Eqs. (3.1) and (3.2) results, respectively, in

$$I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} \tag{3.5}$$

$$I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3} \tag{3.6}$$

The number of nonreference nodes is equal to the number of independent equations that we will derive.

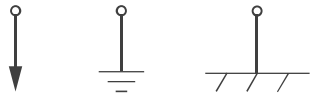
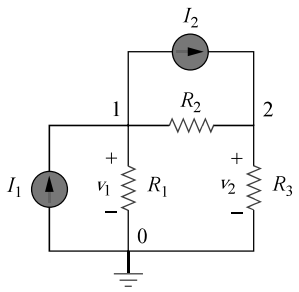
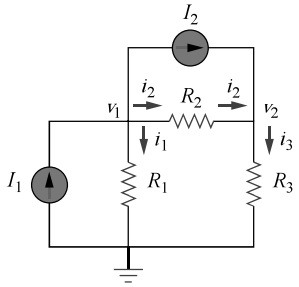


Figure 3.1 Common symbols for indicating a reference node.



(a)



(b)

Figure 3.2 Typical circuit for nodal analysis.

In terms of the conductances, Eqs. (3.5) and (3.6) become

$$I_1 = I_2 + G_1 v_1 + G_2(v_1 - v_2) \tag{3.7}$$

$$I_2 + G_2(v_1 - v_2) = G_3 v_2 \tag{3.8}$$

The third step in nodal analysis is to solve for the node voltages. If we apply KCL to $n-1$ nonreference nodes, we obtain $n-1$ simultaneous equations such as Eqs. (3.5) and (3.6) or (3.7) and (3.8). For the circuit of Fig. 3.2, we solve Eqs. (3.5) and (3.6) or (3.7) and (3.8) to obtain the node voltages v_1 and v_2 using any standard method, such as the substitution method, the elimination method, Cramer's rule, or matrix inversion. To use either of the last two methods, one must cast the simultaneous equations in matrix form. For example, Eqs. (3.7) and (3.8) can be cast in matrix form as

Appendix A discusses how to use Cramer's rule.

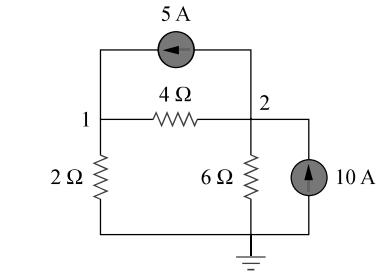
$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix} \tag{3.9}$$

which can be solved to get v_1 and v_2. Equation 3.9 will be generalized in Section 3.6. The simultaneous equations may also be solved using calculators such as HP48 or with software packages such as *Matlab*, *Mathcad*, *Maple*, and *Quattro Pro*.

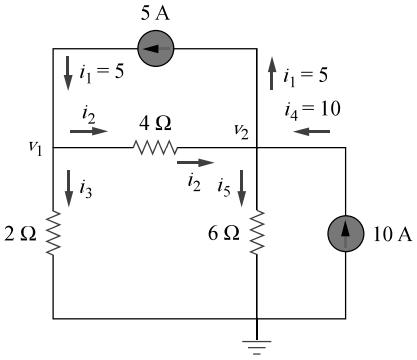
EXAMPLE 3.1

Calculate the node voltages in the circuit shown in Fig. 3.3(a).

Solution:



(a)



(b)

Figure 3.3 For Example 3.1: (a) original circuit, (b) circuit for analysis.

Consider Fig. 3.3(b), where the circuit in Fig. 3.3(a) has been prepared for nodal analysis. Notice how the currents are selected for the application of KCL. Except for the branches with current sources, the labeling of the currents is arbitrary but consistent. (By consistent, we mean that if, for example, we assume that i_2 enters the 4 Ω resistor from the left-hand side, i_2 must leave the resistor from the right-hand side.) The reference node is selected, and the node voltages v_1 and v_2 are now to be determined.

At node 1, applying KCL and Ohm's law gives

$$i_1 = i_2 + i_3 \quad \Longrightarrow \quad 5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2}$$

Multiplying each term in the last equation by 4, we obtain

$$20 = v_1 - v_2 + 2v_1$$

or

$$3v_1 - v_2 = 20 \tag{3.1.1}$$

At node 2, we do the same thing and get

$$i_2 + i_4 = i_1 + i_5 \quad \Longrightarrow \quad \frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2 - 0}{6}$$

Multiplying each term by 12 results in

$$3v_1 - 3v_2 + 120 = 60 + 2v_2$$

or

$$-3v_1 + 5v_2 = 60 \tag{3.1.2}$$

Now we have two simultaneous Eqs. (3.1.1) and (3.1.2). We can solve the equations using any method and obtain the values of v_1 and v_2.

METHOD 1 Using the elimination technique, we add Eqs. (3.1.1) and (3.1.2).

$$4v_2 = 80 \quad \Longrightarrow \quad v_2 = 20 \text{ V}$$

Substituting $v_2 = 20$ in Eq. (3.1.1) gives

$$3v_1 - 20 = 20 \quad \Longrightarrow \quad v_1 = \frac{40}{3} = 13.33 \text{ V}$$

METHOD 2 To use Cramer's rule, we need to put Eqs. (3.1.1) and (3.1.2) in matrix form as

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix} \tag{3.1.3}$$

The determinant of the matrix is

$$\Delta = \begin{vmatrix} 3 & -1 \\ -3 & 5 \end{vmatrix} = 15 - 3 = 12$$

We now obtain v_1 and v_2 as

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 20 & -1 \\ 60 & 5 \end{vmatrix}}{\Delta} = \frac{100 + 60}{12} = 13.33 \text{ V}$$

$$v_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 3 & 20 \\ -3 & 60 \end{vmatrix}}{\Delta} = \frac{180 + 60}{12} = 20 \text{ V}$$

giving us the same result as did the elimination method.

If we need the currents, we can easily calculate them from the values of the nodal voltages.

$$i_1 = 5 \text{ A}, \qquad i_2 = \frac{v_1 - v_2}{4} = -1.6667 \text{ A}, \qquad i_3 = \frac{v_1}{2} = 6.666$$

$$i_4 = 10 \text{ A}, \qquad i_5 = \frac{v_2}{6} = 3.333 \text{ A}$$

The fact that i_2 is negative shows that the current flows in the direction opposite to the one assumed.

PRACTICE PROBLEM 3.1

Obtain the node voltages in the circuit in Fig. 3.4.

Answer: $v_1 = -2$ V, $v_2 = -14$ V.

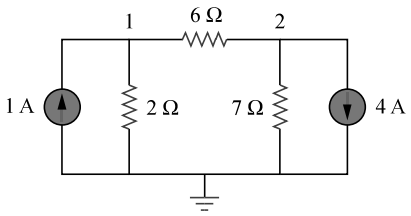


Figure 3.4 For Practice Prob. 3.1.

EXAMPLE 3.2

Determine the voltages at the nodes in Fig. 3.5(a).

Solution:

The circuit in this example has three nonreference nodes, unlike the previous example which has two nonreference nodes. We assign voltages to the three nodes as shown in Fig. 3.5(b) and label the currents.

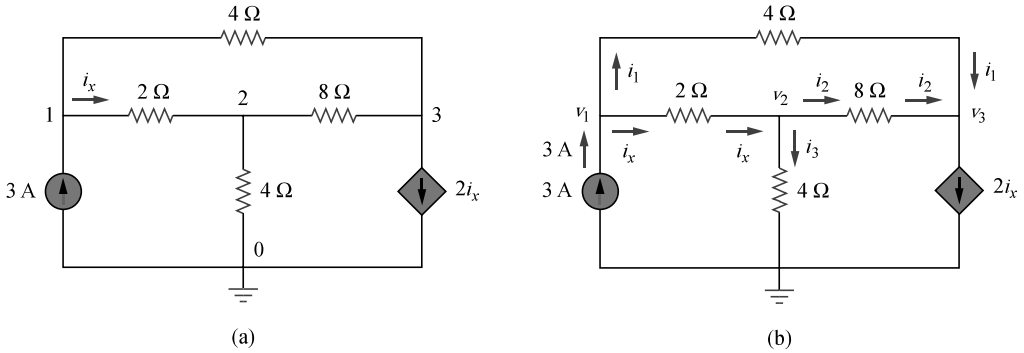


Figure 3.5 For Example 3.2: (a) original circuit, (b) circuit for analysis.

At node 1,

$$3 = i_1 + i_x \qquad \Longrightarrow \qquad 3 = \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2}$$

Multiplying by 4 and rearranging terms, we get

$$3v_1 - 2v_2 - v_3 = 12 \tag{3.2.1}$$

At node 2,

$$i_x = i_2 + i_3 \qquad \Longrightarrow \qquad \frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4}$$

Multiplying by 8 and rearranging terms, we get

$$-4v_1 + 7v_2 - v_3 = 0 \tag{3.2.2}$$

At node 3,

$$i_1 + i_2 = 2i_x \qquad \Longrightarrow \qquad \frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_1 - v_2)}{2}$$

Multiplying by 8, rearranging terms, and dividing by 3, we get

$$2v_1 - 3v_2 + v_3 = 0 \tag{3.2.3}$$

We have three simultaneous equations to solve to get the node voltages v_1, v_2, and v_3. We shall solve the equations in two ways.

METHOD 1 Using the elimination technique, we add Eqs. (3.2.1) and (3.2.3).

$$5v_1 - 5v_2 = 12$$

or

$$v_1 - v_2 = \frac{12}{5} = 2.4 \tag{3.2.4}$$

Adding Eqs. (3.2.2) and (3.2.3) gives

$$-2v_1 + 4v_2 = 0 \quad \Longrightarrow \quad v_1 = 2v_2 \tag{3.2.5}$$

Substituting Eq. (3.2.5) into Eq. (3.2.4) yields

$$2v_2 - v_2 = 2.4 \quad \Longrightarrow \quad v_2 = 2.4, \quad v_1 = 2v_2 = 4.8 \text{ V}$$

From Eq. (3.2.3), we get

$$v_3 = 3v_2 - 2v_1 = 3v_2 - 4v_2 = -v_2 = -2.4 \text{ V}$$

Thus,

$$v_1 = 4.8 \text{ V}, \quad v_2 = 2.4 \text{ V}, \quad v_3 = -2.4 \text{ V}$$

METHOD 2 To use Cramer's rule, we put Eqs. (3.2.1) to (3.2.3) in matrix form.

$$\begin{bmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

From this, we obtain

$$v_1 = \frac{\Delta_1}{\Delta}, \quad v_2 = \frac{\Delta_2}{\Delta}, \quad v_3 = \frac{\Delta_3}{\Delta}$$

where Δ, Δ_1, Δ_2, and Δ_3 are the determinants to be calculated as follows. As explained in Appendix A, to calculate the determinant of a 3 by 3 matrix, we repeat the first two rows and cross multiply.

$$\Delta = \begin{vmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{vmatrix} = \begin{array}{ccc} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \\ 3 & -2 & -1 \\ -4 & 7 & -1 \end{array} \quad \begin{array}{l} - \\ - \\ - \\ + \\ + \\ + \end{array}$$

$$= 21 - 12 + 4 + 14 - 9 - 8 = 10$$

Similarly, we obtain

$$\Delta_1 = \begin{array}{ccc} 12 & -2 & -1 \\ 0 & 7 & -1 \\ 0 & -3 & 1 \\ 12 & -2 & -1 \\ 0 & 7 & -1 \end{array} \quad \begin{array}{l} - \\ - \\ - \\ + \\ + \\ + \end{array} \quad = 84 + 0 + 0 - 0 - 36 - 0 = 48$$

$$\Delta_2 = \begin{vmatrix} 3 & 12 & -1 \\ -4 & 0 & -1 \\ 2 & 0 & 1 \\ 3 & 12 & -1 \\ -4 & 0 & -1 \end{vmatrix} = 0 + 0 - 24 - 0 - 0 + 48 = 24$$

$$\Delta_3 = \begin{vmatrix} 3 & -2 & 12 \\ -4 & 7 & 0 \\ 2 & -3 & 0 \\ 3 & -2 & 12 \\ -4 & 7 & 0 \end{vmatrix} = 0 + 144 + 0 - 168 - 0 - 0 = -24$$

Thus, we find

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{48}{10} = 4.8 \text{ V}, \qquad v_2 = \frac{\Delta_2}{\Delta} = \frac{24}{10} = 2.4 \text{ V}$$

$$v_3 = \frac{\Delta_3}{\Delta} = \frac{-24}{10} = -2.4 \text{ V}$$

as we obtained with Method 1.

PRACTICE PROBLEM 3.2

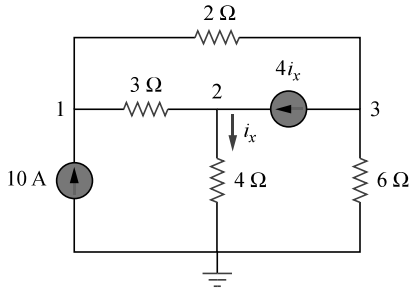


Figure 3.6 For Practice Prob. 3.2.

Find the voltages at the three nonreference nodes in the circuit of Fig. 3.6.

Answer: $v_1 = 80$ V, $v_2 = -64$ V, $v_3 = 156$ V.

Electronic Testing Tutorials

3.3 NODAL ANALYSIS WITH VOLTAGE SOURCES

We now consider how voltage sources affect nodal analysis. We use the circuit in Fig. 3.7 for illustration. Consider the following two possibilities.

CASE 1 If a voltage source is connected between the reference node and a nonreference node, we simply set the voltage at the nonreference node equal to the voltage of the voltage source. In Fig. 3.7, for example,

$$v_1 = 10 \text{ V} \tag{3.10}$$

Thus our analysis is somewhat simplified by this knowledge of the voltage at this node.

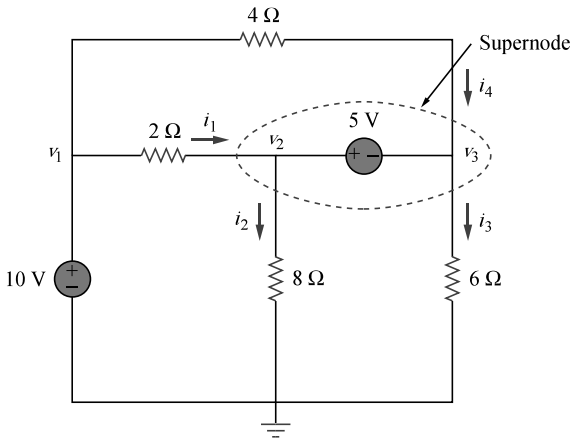


Figure 3.7 A circuit with a supernode.

CASE 2 If the voltage source (dependent or independent) is connected between two nonreference nodes, the two nonreference nodes form a *generalized node* or *supernode*; we apply both KCL and KVL to determine the node voltages.

A supernode may be regarded as a closed surface enclosing the voltage source and its two nodes.

> A supernode is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.

In Fig. 3.7, nodes 2 and 3 form a supernode. (We could have more than two nodes forming a single supernode. For example, see the circuit in Fig. 3.14.) We analyze a circuit with supernodes using the same three steps mentioned in the previous section except that the supernodes are treated differently. Why? Because an essential component of nodal analysis is applying KCL, which requires knowing the current through each element. There is no way of knowing the current through a voltage source in advance. However, KCL must be satisfied at a supernode like any other node. Hence, at the supernode in Fig. 3.7,

$$i_1 + i_4 = i_2 + i_3 \tag{3.11a}$$

or

$$\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = \frac{v_2 - 0}{8} + \frac{v_3 - 0}{6} \tag{3.11b}$$

To apply Kirchhoff's voltage law to the supernode in Fig. 3.7, we redraw the circuit as shown in Fig. 3.8. Going around the loop in the clockwise direction gives

$$-v_2 + 5 + v_3 = 0 \quad \Longrightarrow \quad v_2 - v_3 = 5 \tag{3.12}$$

From Eqs. (3.10), (3.11b), and (3.12), we obtain the node voltages.

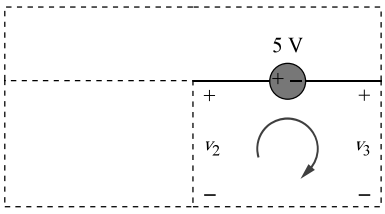


Figure 3.8 Applying KVL to a supernode.

Note the following properties of a supernode:

1. The voltage source inside the supernode provides a constraint equation needed to solve for the node voltages.
2. A supernode has no voltage of its own.
3. A supernode requires the application of both KCL and KVL.

EXAMPLE 3.3

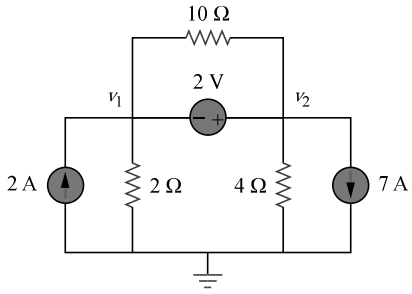


Figure 3.9 For Example 3.3.

For the circuit shown in Fig. 3.9, find the node voltages.

Solution:

The supernode contains the 2-V source, nodes 1 and 2, and the 10-Ω resistor. Applying KCL to the supernode as shown in Fig. 3.10(a) gives

$$2 = i_1 + i_2 + 7$$

Expressing i_1 and i_2 in terms of the node voltages

$$2 = \frac{v_1 - 0}{2} + \frac{v_2 - 0}{4} + 7 \qquad \Longrightarrow \qquad 8 = 2v_1 + v_2 + 28$$

or

$$v_2 = -20 - 2v_1 \tag{3.3.1}$$

To get the relationship between v_1 and v_2, we apply KVL to the circuit in Fig. 3.10(b). Going around the loop, we obtain

$$-v_1 - 2 + v_2 = 0 \qquad \Longrightarrow \qquad v_2 = v_1 + 2 \tag{3.3.2}$$

From Eqs. (3.3.1) and (3.3.2), we write

$$v_2 = v_1 + 2 = -20 - 2v_1$$

or

$$3v_1 = -22 \qquad \Longrightarrow \qquad v_1 = -7.333 \text{ V}$$

and $v_2 = v_1 + 2 = -5.333$ V. Note that the 10-Ω resistor does not make any difference because it is connected across the supernode.

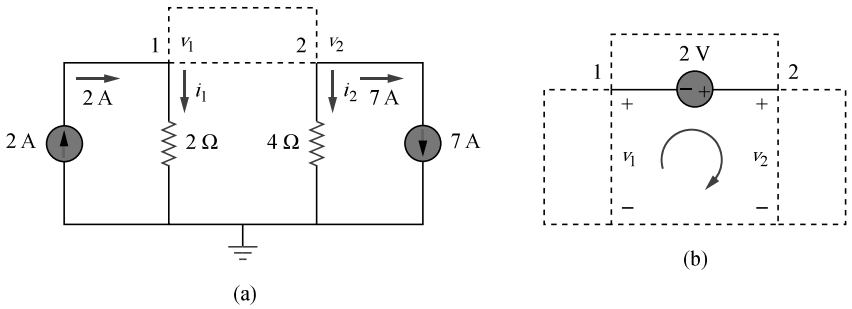


Figure 3.10 Applying: (a) KCL to the supernode, (b) KVL to the loop.

PRACTICE PROBLEM 3.3

Find v and i in the circuit in Fig. 3.11.

Answer: -0.2 V, 1.4 A.

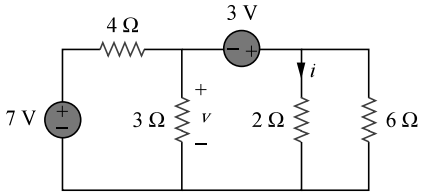


Figure 3.11 For Practice Prob. 3.3.

EXAMPLE 3.4

Find the node voltages in the circuit of Fig. 3.12.

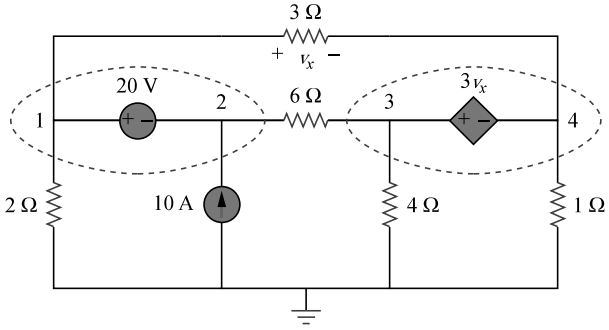


Figure 3.12 For Example 3.4.

Solution:

Nodes 1 and 2 form a supernode; so do nodes 3 and 4. We apply KCL to the two supernodes as in Fig. 3.13(a). At supernode 1-2,

$$i_3 + 10 = i_1 + i_2$$

Expressing this in terms of the node voltages,

$$\frac{v_3 - v_2}{6} + 10 = \frac{v_1 - v_4}{3} + \frac{v_1}{2}$$

or

$$5v_1 + v_2 - v_3 - 2v_4 = 60 \tag{3.4.1}$$

At supernode 3-4,

$$i_1 = i_3 + i_4 + i_5 \qquad \Longrightarrow \qquad \frac{v_1 - v_4}{3} = \frac{v_3 - v_2}{6} + \frac{v_4}{1} + \frac{v_3}{4}$$

or

$$4v_1 + 2v_2 - 5v_3 - 16v_4 = 0 \tag{3.4.2}$$

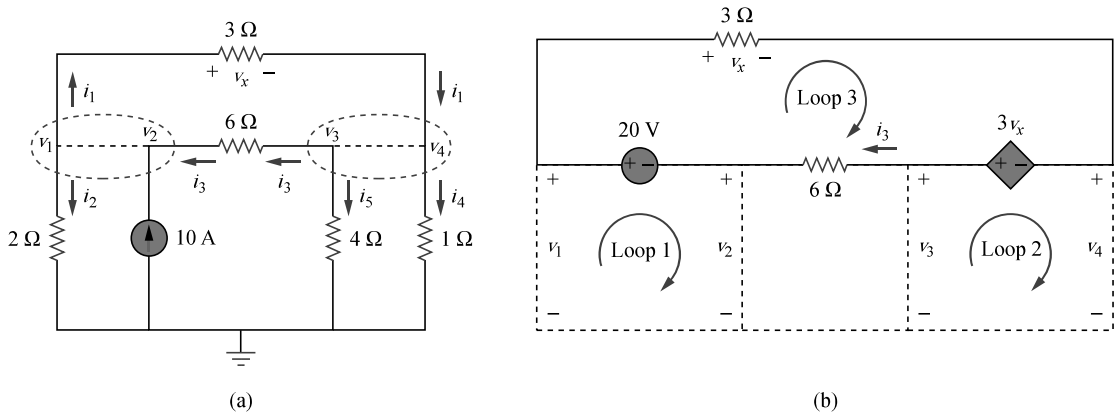


Figure 3.13 Applying: (a) KCL to the two supernodes, (b) KVL to the loops.

We now apply KVL to the branches involving the voltage sources as shown in Fig. 3.13(b). For loop 1,

$$-v_1 + 20 + v_2 = 0 \quad \Longrightarrow \quad v_1 - v_2 = 20 \tag{3.4.3}$$

For loop 2,

$$-v_3 + 3v_x + v_4 = 0$$

But $v_x = v_1 - v_4$ so that

$$3v_1 - v_3 - 2v_4 = 0 \tag{3.4.4}$$

For loop 3,

$$v_x - 3v_x + 6i_3 - 20 = 0$$

But $6i_3 = v_3 - v_2$ and $v_x = v_1 - v_4$. Hence

$$-2v_1 - v_2 + v_3 + 2v_4 = 20 \tag{3.4.5}$$

We need four node voltages, v_1, v_2, v_3, and v_4, and it requires only four out of the five Eqs. (3.4.1) to (3.4.5) to find them. Although the fifth equation is redundant, it can be used to check results. We can eliminate one node voltage so that we solve three simultaneous equations instead of four. From Eq. (3.4.3), $v_2 = v_1 - 20$. Substituting this into Eqs. (3.4.1) and (3.4.2), respectively, gives

$$6v_1 - v_3 - 2v_4 = 80 \tag{3.4.6}$$

and

$$6v_1 - 5v_3 - 16v_4 = 40 \tag{3.4.7}$$

Equations (3.4.4), (3.4.6), and (3.4.7) can be cast in matrix form as

$$\begin{bmatrix} 3 & -1 & -2 \\ 6 & -1 & -2 \\ 6 & -5 & -16 \end{bmatrix} \begin{bmatrix} v_1 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 80 \\ 40 \end{bmatrix}$$

Using Cramer's rule,

$$\Delta = \begin{vmatrix} 3 & -1 & -2 \\ 6 & -1 & -2 \\ 6 & -5 & -16 \end{vmatrix} = -18, \qquad \Delta_1 = \begin{vmatrix} 0 & -1 & -2 \\ 80 & -1 & -2 \\ 40 & -5 & -16 \end{vmatrix} = -480$$

$$\Delta_3 = \begin{vmatrix} 3 & 0 & -2 \\ 6 & 80 & -2 \\ 6 & 40 & -16 \end{vmatrix} = -3120, \qquad \Delta_4 = \begin{vmatrix} 3 & -1 & 0 \\ 6 & -1 & 80 \\ 6 & -5 & 40 \end{vmatrix} = 840$$

Thus, we arrive at the node voltages as

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{-480}{-18} = 26.667 \text{ V}, \qquad v_3 = \frac{\Delta_3}{\Delta} = \frac{-3120}{-18} = 173.333 \text{ V}$$

$$v_4 = \frac{\Delta_4}{\Delta} = \frac{840}{-18} = -46.667 \text{ V}$$

and $v_2 = v_1 - 20 = 6.667$ V. We have not used Eq. (3.4.5); it can be used to cross check results.



PRACTICE PROBLEM 3.4

Find v_1, v_2, and v_3 in the circuit in Fig. 3.14 using nodal analysis.

Answer: $v_1 = 3.043$ V, $v_2 = -6.956$ V, $v_3 = 0.6522$ V.

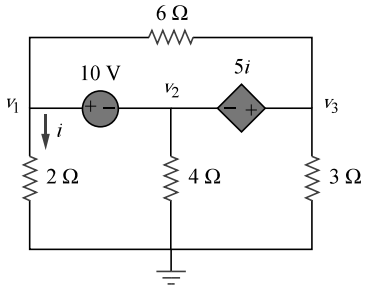
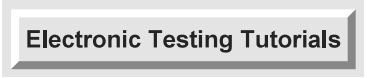


Figure 3.14 For Practice Prob. 3.4.

3.4 MESH ANALYSIS



Mesh analysis provides another general procedure for analyzing circuits, using mesh currents as the circuit variables. Using mesh currents instead of element currents as circuit variables is convenient and reduces the number of equations that must be solved simultaneously. Recall that a loop is a closed path with no node passed more than once. A mesh is a loop that does not contain any other loop within it.

Nodal analysis applies KCL to find unknown voltages in a given circuit, while mesh analysis applies KVL to find unknown currents. Mesh analysis is not quite as general as nodal analysis because it is only applicable to a circuit that is *planar*. A planar circuit is one that can be drawn in a plane with no branches crossing one another; otherwise it is *nonplanar*. A circuit may have crossing branches and still be planar if it can be redrawn such that it has no crossing branches. For example, the

Mesh analysis is also known as *loop analysis* or the *mesh-current method.*

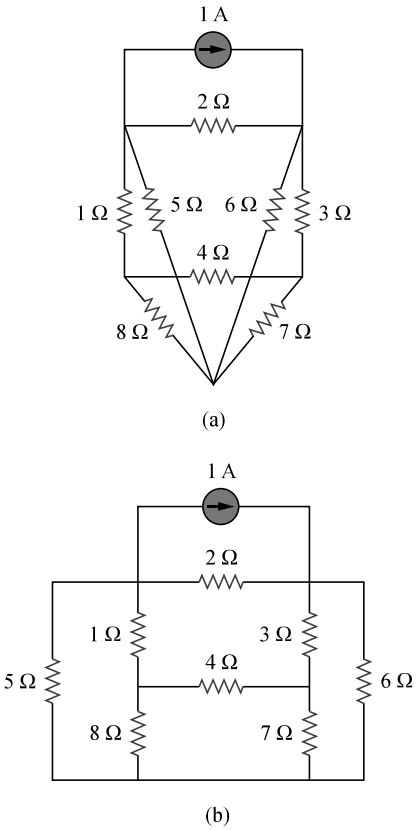


Figure 3.15 (a) A planar circuit with crossing branches, (b) the same circuit redrawn with no crossing branches.

circuit in Fig. 3.15(a) has two crossing branches, but it can be redrawn as in Fig. 3.15(b). Hence, the circuit in Fig. 3.15(a) is planar. However, the circuit in Fig. 3.16 is nonplanar, because there is no way to redraw it and avoid the branches crossing. Nonplanar circuits can be handled using nodal analysis, but they will not be considered in this text.

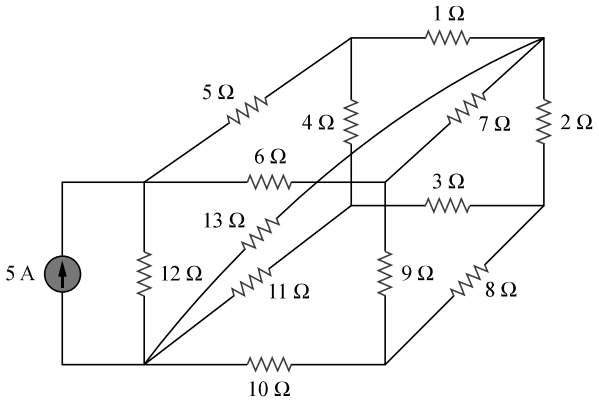


Figure 3.16 A nonplanar circuit.

To understand mesh analysis, we should first explain more about what we mean by a mesh.

A mesh is a loop which does not contain any other loops within it.

In Fig. 3.17, for example, paths *abefa* and *bcdeb* are meshes, but path *abcdefa* is not a mesh. The current through a mesh is known as *mesh current.* In mesh analysis, we are interested in applying KVL to find the mesh currents in a given circuit.

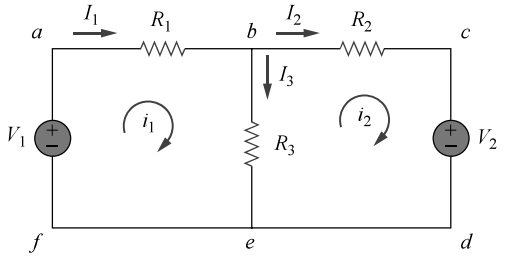


Figure 3.17 A circuit with two meshes.

In this section, we will apply mesh analysis to planar circuits that do not contain current sources. In the next sections, we will consider circuits with current sources. In the mesh analysis of a circuit with n meshes, we take the following three steps.

Although path *abcdefa* is a loop and not a mesh, KVL still holds. This is the reason for loosely using the terms *loop analysis* and *mesh analysis* to mean the same thing.

Steps to Determine Mesh Currents:

1. Assign mesh currents $i_1, i_2, \ldots, i_n$ to the n meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

To illustrate the steps, consider the circuit in Fig. 3.17. The first step requires that mesh currents i_1 and i_2 are assigned to meshes 1 and 2. Although a mesh current may be assigned to each mesh in an arbitrary direction, it is conventional to assume that each mesh current flows clockwise.

The direction of the mesh current is arbitrary—(clockwise or counterclockwise)—and does not affect the validity of the solution.

As the second step, we apply KVL to each mesh. Applying KVL to mesh 1, we obtain

$$-V_1 + R_1 i_1 + R_3(i_1 - i_2) = 0$$

or

$$(R_1 + R_3)i_1 - R_3 i_2 = V_1 \tag{3.13}$$

For mesh 2, applying KVL gives

$$R_2 i_2 + V_2 + R_3(i_2 - i_1) = 0$$

or

$$-R_3 i_1 + (R_2 + R_3)i_2 = -V_2 \tag{3.14}$$

Note in Eq. (3.13) that the coefficient of i_1 is the sum of the resistances in the first mesh, while the coefficient of i_2 is the negative of the resistance common to meshes 1 and 2. Now observe that the same is true in Eq. (3.14). This can serve as a shortcut way of writing the mesh equations. We will exploit this idea in Section 3.6.

The shortcut way will not apply if one mesh current is assumed clockwise and the other assumed anticlockwise, although this is permissible.

The third step is to solve for the mesh currents. Putting Eqs. (3.13) and (3.14) in matrix form yields

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix} \tag{3.15}$$

which can be solved to obtain the mesh currents i_1 and i_2. We are at liberty to use any technique for solving the simultaneous equations. According to Eq. (2.12), if a circuit has n nodes, b branches, and l independent loops or meshes, then $l = b - n + 1$. Hence, l independent simultaneous equations are required to solve the circuit using mesh analysis.

Notice that the branch currents are different from the mesh currents unless the mesh is isolated. To distinguish between the two types of currents, we use i for a mesh current and I for a branch current. The current elements I_1, I_2, and I_3 are algebraic sums of the mesh currents. It is evident from Fig. 3.17 that

$$I_1 = i_1, \qquad I_2 = i_2, \qquad I_3 = i_1 - i_2 \tag{3.16}$$

EXAMPLE 3.5

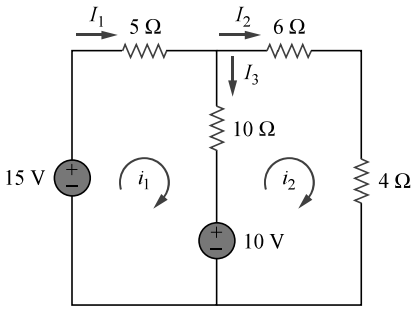


Figure 3.18 For Example 3.5.

For the circuit in Fig. 3.18, find the branch currents I_1, I_2, and I_3 using mesh analysis.

Solution:

We first obtain the mesh currents using KVL. For mesh 1,

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

or

$$3i_1 - 2i_2 = 1 \tag{3.5.1}$$

For mesh 2,

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$$

or

$$i_1 = 2i_2 - 1 \tag{3.5.2}$$

METHOD 1 Using the substitution method, we substitute Eq. (3.5.2) into Eq. (3.5.1), and write

$$6i_2 - 3 - 2i_2 = 1 \qquad \Longrightarrow \qquad i_2 = 1 \text{ A}$$

From Eq. (3.5.2), $i_1 = 2i_2 - 1 = 2 - 1 = 1$ A. Thus,

$$I_1 = i_1 = 1 \text{ A}, \qquad I_2 = i_2 = 1 \text{ A}, \qquad I_3 = i_1 - i_2 = 0$$

METHOD 2 To use Cramer's rule, we cast Eqs. (3.5.1) and (3.5.2) in matrix form as

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

We obtain the determinants

$$\Delta = \begin{vmatrix} 3 & -2 \\ -1 & 2 \end{vmatrix} = 6 - 2 = 4$$

$$\Delta_1 = \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} = 2 + 2 = 4, \qquad \Delta_2 = \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = 3 + 1 = 4$$

Thus,

$$i_1 = \frac{\Delta_1}{\Delta} = 1 \text{ A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = 1 \text{ A}$$

as before.

PRACTICE PROBLEM 3.5

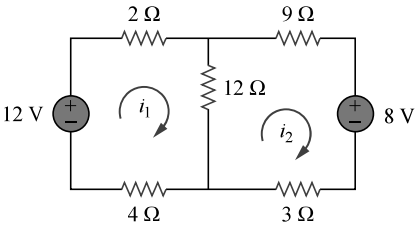


Figure 3.19 For Practice Prob. 3.5.

Calculate the mesh currents i_1 and i_2 in the circuit of Fig. 3.19.

Answer: $i_1 = \frac{2}{3}$ A, $i_2 = 0$ A.

EXAMPLE 3.6

Use mesh analysis to find the current i_o in the circuit in Fig. 3.20.

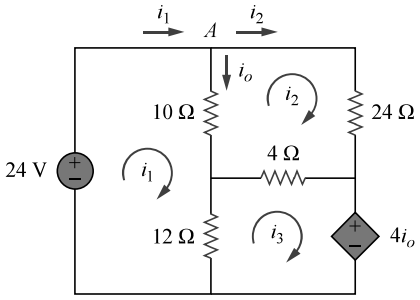


Figure 3.20 For Example 3.6.

Solution:

We apply KVL to the three meshes in turn. For mesh 1,

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$$

or

$$11i_1 - 5i_2 - 6i_3 = 12 \tag{3.6.1}$$

For mesh 2,

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$$

or

$$-5i_1 + 19i_2 - 2i_3 = 0 \tag{3.6.2}$$

For mesh 3,

$$4i_o + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

But at node A, $i_o = i_1 - i_2$, so that

$$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

or

$$-i_1 - i_2 + 2i_3 = 0 \tag{3.6.3}$$

In matrix form, Eqs. (3.6.1) to (3.6.3) become

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

We obtain the determinants as

$$\Delta = \begin{vmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{vmatrix}$$

$$= 418 - 30 - 10 - 114 - 22 - 50 = 192$$

$$\Delta_1 = \begin{vmatrix} 12 & -5 & -6 \\ 0 & 19 & -2 \\ 0 & -1 & 2 \end{vmatrix} = 456 - 24 = 432$$

$$\Delta_2 = \begin{vmatrix} 11 & 12 & -6 \\ -5 & 0 & -2 \\ -1 & 0 & 2 \end{vmatrix} = 24 + 120 = 144$$

$$\Delta_3 = \begin{vmatrix} 11 & -5 & 12 \\ -5 & 19 & 0 \\ -1 & -1 & 0 \end{vmatrix} = 60 + 228 = 288$$

We calculate the mesh currents using Cramer's rule as

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{432}{192} = 2.25 \text{ A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = \frac{144}{192} = 0.75 \text{ A}$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{288}{192} = 1.5 \text{ A}$$

Thus, $i_o = i_1 - i_2 = 1.5$ A.

PRACTICE PROBLEM 3.6

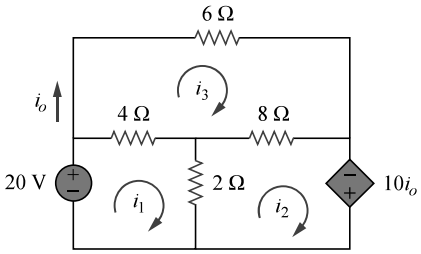


Figure 3.21 For Practice Prob. 3.6.

Using mesh analysis, find i_o in the circuit in Fig. 3.21.

Answer: −5 A.

Electronic Testing Tutorials

3.5 MESH ANALYSIS WITH CURRENT SOURCES

Applying mesh analysis to circuits containing current sources (dependent or independent) may appear complicated. But it is actually much easier than what we encountered in the previous section, because the presence of the current sources reduces the number of equations. Consider the following two possible cases.

CASE 1 When a current source exists only in one mesh: Consider the circuit in Fig. 3.22, for example. We set $i_2 = -5$ A and write a mesh equation for the other mesh in the usual way, that is,

$$-10 + 4i_1 + 6(i_1 - i_2) = 0 \qquad \Longrightarrow \qquad i_1 = -2 \text{ A} \tag{3.17}$$

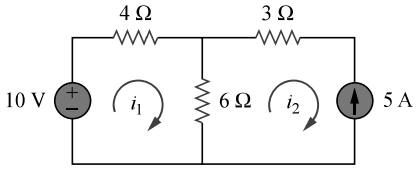


Figure 3.22 A circuit with a current source.

CASE 2 When a current source exists between two meshes: Consider the circuit in Fig. 3.23(a), for example. We create a *supermesh* by excluding the current source and any elements connected in series with it, as shown in Fig. 3.23(b). Thus,

A supermesh results when two meshes have a (dependent or independent) current source in common.

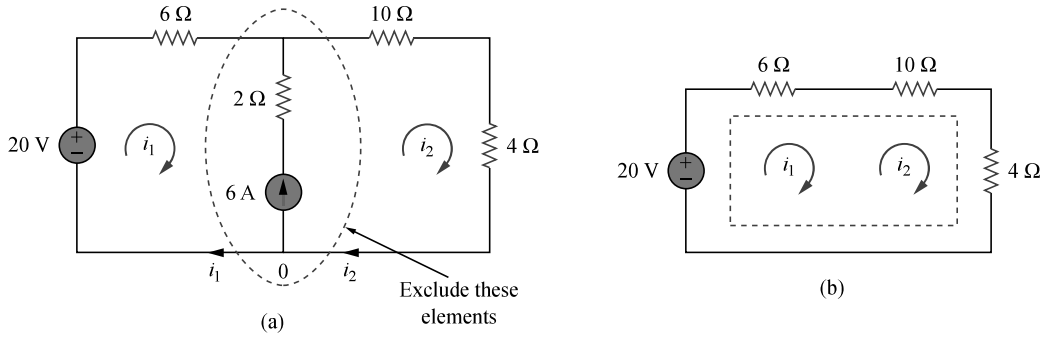


Figure 3.23 (a) Two meshes having a current source in common, (b) a supermesh, created by excluding the current source.

As shown in Fig. 3.23(b), we create a supermesh as the periphery of the two meshes and treat it differently. (If a circuit has two or more supermeshes that intersect, they should be combined to form a larger supermesh.) Why treat the supermesh differently? Because mesh analysis applies KVL—which requires that we know the voltage across each branch—and we do not know the voltage across a current source in advance. However, a supermesh must satisfy KVL like any other mesh. Therefore, applying KVL to the supermesh in Fig. 3.23(b) gives

$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$

or

$$6i_1 + 14i_2 = 20 \tag{3.18}$$

We apply KCL to a node in the branch where the two meshes intersect. Applying KCL to node 0 in Fig. 3.23(a) gives

$$i_2 = i_1 + 6 \tag{3.19}$$

Solving Eqs. (3.18) and (3.19), we get

$$i_1 = -3.2 \text{ A}, \qquad i_2 = 2.8 \text{ A} \tag{3.20}$$

Note the following properties of a supermesh:

1. The current source in the supermesh is not completely ignored; it provides the constraint equation necessary to solve for the mesh currents.
2. A supermesh has no current of its own.
3. A supermesh requires the application of both KVL and KCL.

EXAMPLE 3.7

For the circuit in Fig. 3.24, find i_1 to i_4 using mesh analysis.

Solution:

Note that meshes 1 and 2 form a supermesh since they have an independent current source in common. Also, meshes 2 and 3 form another supermesh

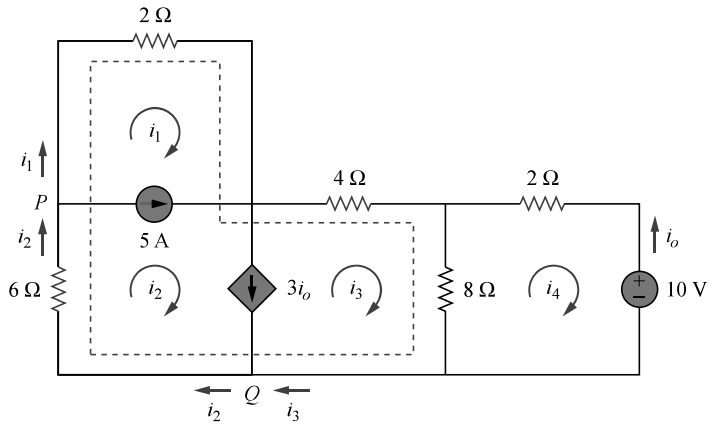


Figure 3.24 For Example 3.7.

because they have a dependent current source in common. The two supermeshes intersect and form a larger supermesh as shown. Applying KVL to the larger supermesh,

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

or

$$i_1 + 3i_2 + 6i_3 - 4i_4 = 0 \tag{3.7.1}$$

For the independent current source, we apply KCL to node P:

$$i_2 = i_1 + 5 \tag{3.7.2}$$

For the dependent current source, we apply KCL to node Q:

$$i_2 = i_3 + 3i_o$$

But $i_o = -i_4$, hence,

$$i_2 = i_3 - 3i_4 \tag{3.7.3}$$

Applying KVL in mesh 4,

$$2i_4 + 8(i_4 - i_3) + 10 = 0$$

or

$$5i_4 - 4i_3 = -5 \tag{3.7.4}$$

From Eqs. (3.7.1) to (3.7.4),

$$i_1 = -7.5 \text{ A}, \quad i_2 = -2.5 \text{ A}, \quad i_3 = 3.93 \text{ A}, \quad i_4 = 2.143 \text{ A}$$

PRACTICE PROBLEM 3.7

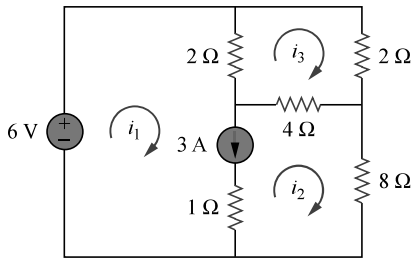


Figure 3.25 For Practice Prob. 3.7.

Use mesh analysis to determine i_1, i_2, and i_3 in Fig. 3.25.

Answer: $i_1 = 3.474$ A, $i_2 = 0.4737$ A, $i_3 = 1.1052$ A.

@ Network Analysis

†3.6 NODAL AND MESH ANALYSES BY INSPECTION

This section presents a generalized procedure for nodal or mesh analysis. It is a shortcut approach based on mere inspection of a circuit.

When all sources in a circuit are independent current sources, we do not need to apply KCL to each node to obtain the node-voltage equations as we did in Section 3.2. We can obtain the equations by mere inspection of the circuit. As an example, let us reexamine the circuit in Fig. 3.2, shown again in Fig. 3.26(a) for convenience. The circuit has two nonreference nodes and the node equations were derived in Section 3.2 as

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix} \tag{3.21}$$

Observe that each of the diagonal terms is the sum of the conductances connected directly to node 1 or 2, while the off-diagonal terms are the negatives of the conductances connected between the nodes. Also, each term on the right-hand side of Eq. (3.21) is the algebraic sum of the currents entering the node.

In general, if a circuit with independent current sources has N nonreference nodes, the node-voltage equations can be written in terms of the conductances as

$$\begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1N} \\ G_{21} & G_{22} & \cdots & G_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ G_{N1} & G_{N2} & \cdots & G_{NN} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} \tag{3.22}$$

or simply

$$\mathbf{Gv} = \mathbf{i} \tag{3.23}$$

where

G_{kk} = Sum of the conductances connected to node k

$G_{kj} = G_{jk}$ = Negative of the sum of the conductances directly connecting nodes k and j, $k \neq j$

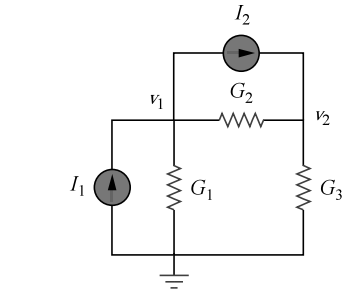
v_k = Unknown voltage at node k

i_k = Sum of all independent current sources directly connected to node k, with currents entering the node treated as positive

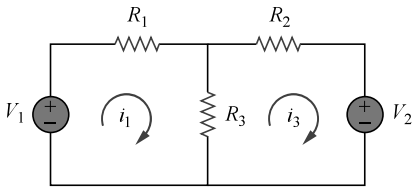
G is called the *conductance matrix*, **v** is the output vector; and **i** is the input vector. Equation (3.22) can be solved to obtain the unknown node voltages. Keep in mind that this is valid for circuits with only independent current sources and linear resistors.

Similarly, we can obtain mesh-current equations by inspection when a linear resistive circuit has only independent voltage sources. Consider the circuit in Fig. 3.17, shown again in Fig. 3.26(b) for convenience. The circuit has two nonreference nodes and the node equations were derived in Section 3.4 as

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ -v_2 \end{bmatrix} \tag{3.24}$$



(a)



(b)

Figure 3.26 (a) The circuit in Fig. 3.2, (b) the circuit in Fig. 3.17.

We notice that each of the diagonal terms is the sum of the resistances in the related mesh, while each of the off-diagonal terms is the negative of the resistance common to meshes 1 and 2. Each term on the right-hand side of Eq. (3.24) is the algebraic sum taken clockwise of all independent voltage sources in the related mesh.

In general, if the circuit has N meshes, the mesh-current equations can be expressed in terms of the resistances as

$$\begin{bmatrix} R_{11} & R_{12} & \dots & R_{1N} \\ R_{21} & R_{22} & \dots & R_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ R_{N1} & R_{N2} & \dots & R_{NN} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} \tag{3.25}$$

or simply

$$\mathbf{Ri} = \mathbf{v} \tag{3.26}$$

where

R_{kk} = Sum of the resistances in mesh k

$R_{kj} = R_{jk}$ = Negative of the sum of the resistances in common with meshes k and j, $k \neq j$

i_k = Unknown mesh current for mesh k in the clockwise direction

v_k = Sum taken clockwise of all independent voltage sources in mesh k, with voltage rise treated as positive

R is called the *resistance matrix*, **i** is the output vector; and **v** is the input vector. We can solve Eq. (3.25) to obtain the unknown mesh currents.

EXAMPLE 3.8

Write the node-voltage matrix equations for the circuit in Fig. 3.27 by inspection.

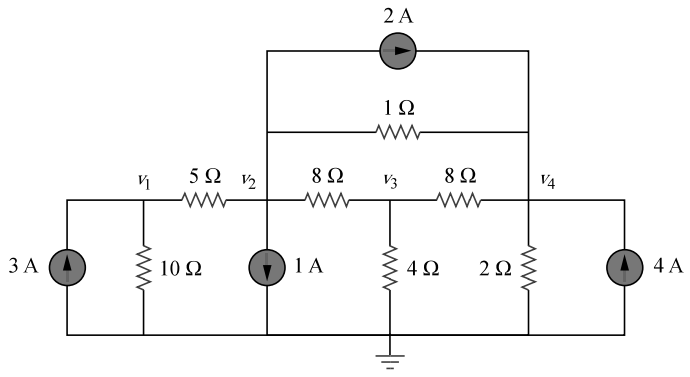


Figure 3.27 For Example 3.8.

Solution:

The circuit in Fig. 3.27 has four nonreference nodes, so we need four node equations. This implies that the size of the conductance matrix **G**, is 4 by 4. The diagonal terms of **G**, in siemens, are

$$G_{11} = \frac{1}{5} + \frac{1}{10} = 0.3, \qquad G_{22} = \frac{1}{5} + \frac{1}{8} + \frac{1}{1} = 1.325$$

$$G_{33} = \frac{1}{8} + \frac{1}{8} + \frac{1}{4} = 0.5, \qquad G_{44} = \frac{1}{8} + \frac{1}{2} + \frac{1}{1} = 1.625$$

The off-diagonal terms are

$$G_{12} = -\frac{1}{5} = -0.2, \qquad G_{13} = G_{14} = 0$$

$$G_{21} = -0.2, \qquad G_{23} = -\frac{1}{8} = -0.125, \qquad G_{24} = -\frac{1}{1} = -1$$

$$G_{31} = 0, \qquad G_{32} = -0.125, \qquad G_{34} = -\frac{1}{8} = -0.125$$

$$G_{41} = 0, \qquad G_{42} = -1, \qquad G_{43} = -0.125$$

The input current vector **i** has the following terms, in amperes:

$$i_1 = 3, \qquad i_2 = -1 - 2 = -3, \qquad i_3 = 0, \qquad i_4 = 2 + 4 = 6$$

Thus the node-voltage equations are

$$\begin{bmatrix} 0.3 & -0.2 & 0 & 0 \\ -0.2 & 1.325 & -0.125 & -1 \\ 0 & -0.125 & 0.5 & -0.125 \\ 0 & -1 & -0.125 & 1.625 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 0 \\ 6 \end{bmatrix}$$

which can be solved to obtain the node voltages v_1, v_2, v_3, and v_4.

PRACTICE PROBLEM 3.8

By inspection, obtain the node-voltage equations for the circuit in Fig. 3.28.

Answer:

$$\begin{bmatrix} 1.3 & -0.2 & -1 & 0 \\ -0.2 & 0.2 & 0 & 0 \\ -1 & 0 & 1.25 & -0.25 \\ 0 & 0 & -0.25 & 0.75 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ -1 \\ 3 \end{bmatrix}$$

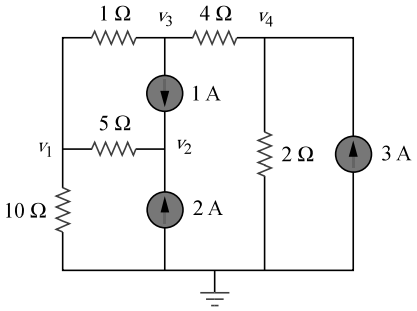


Figure 3.28 For Practice Prob. 3.8.

EXAMPLE 3.9

By inspection, write the mesh-current equations for the circuit in Fig. 3.29.

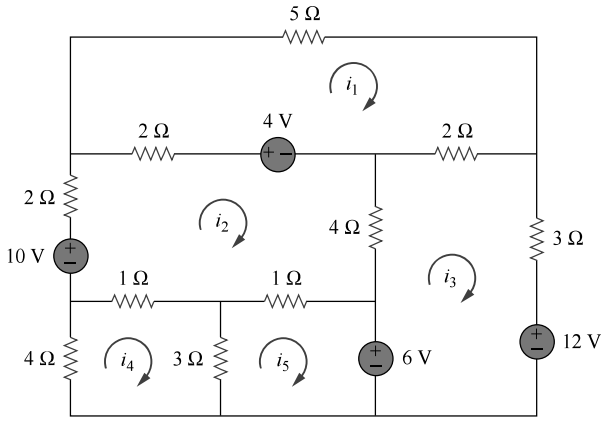


Figure 3.29 For Example 3.9.

Solution:

We have five meshes, so the resistance matrix is 5 by 5. The diagonal terms, in ohms, are:

$$R_{11} = 5 + 2 + 2 = 9, \qquad R_{22} = 2 + 4 + 1 + 1 + 2 = 10$$
$$R_{33} = 2 + 3 + 4 = 9, \qquad R_{44} = 1 + 3 + 4 = 8, \qquad R_{55} = 1 + 3 = 4$$

The off-diagonal terms are:

$$R_{12} = -2, \qquad R_{13} = -2, \qquad R_{14} = 0 = R_{15}$$
$$R_{21} = -2, \qquad R_{23} = -4, \qquad R_{24} = -1, \qquad R_{25} = -1$$
$$R_{31} = -2, \qquad R_{32} = -4, \qquad R_{34} = 0 = R_{35}$$
$$R_{41} = 0, \qquad R_{42} = -1, \qquad R_{43} = 0, \qquad R_{45} = -3$$
$$R_{51} = 0, \qquad R_{52} = -1, \qquad R_{53} = 0, \qquad R_{54} = -3$$

The input voltage vector $\mathbf{v}$ has the following terms in volts:

$$v_1 = 4, \qquad v_2 = 10 - 4 = 6$$
$$v_3 = -12 + 6 = -6, \qquad v_4 = 0, \qquad v_5 = -6$$

Thus the mesh-current equations are:

$$\begin{bmatrix} 9 & -2 & -2 & 0 & 0 \\ -2 & 10 & -4 & -1 & -1 \\ -2 & -4 & 9 & 0 & 0 \\ 0 & -1 & 0 & 8 & -3 \\ 0 & -1 & 0 & -3 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ -6 \\ 0 \\ -6 \end{bmatrix}$$

From this, we can obtain mesh currents i_1, i_2, i_3, i_4, and i_5.

PRACTICE PROBLEM 3.9

By inspection, obtain the mesh-current equations for the circuit in Fig. 3.30.

PROBLEMS

Sections 3.2 and 3.3 Nodal Analysis

3.1 Determine v_1, v_2, and the power dissipated in all the resistors in the circuit of Fig. 3.50.

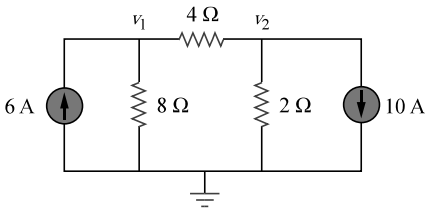


Figure 3.50 For Prob. 3.1.

3.2 For the circuit in Fig. 3.51, obtain v_1 and v_2.

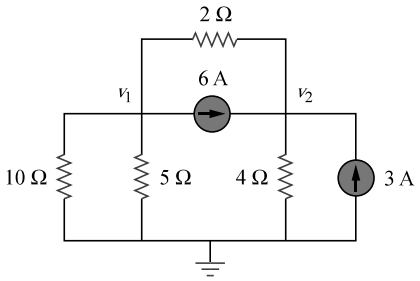


Figure 3.51 For Prob. 3.2.

3.3 Find the currents i_1 through i_4 and the voltage v_o in the circuit in Fig. 3.52.

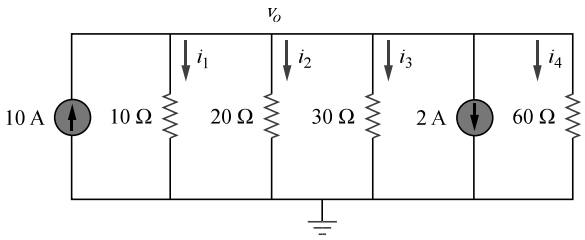


Figure 3.52 For Prob. 3.3.

3.4 Given the circuit in Fig. 3.53, calculate the currents i_1 through i_4.

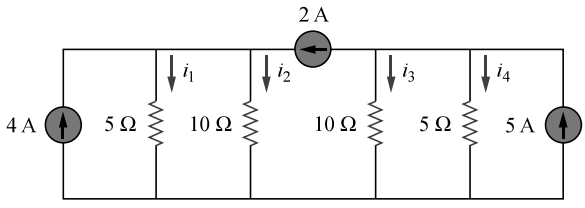


Figure 3.53 For Prob. 3.4.

3.5 Obtain v_o in the circuit of Fig. 3.54.

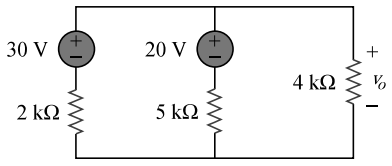


Figure 3.54 For Prob. 3.5.

3.6 Use nodal analysis to obtain v_o in the circuit in Fig. 3.55.

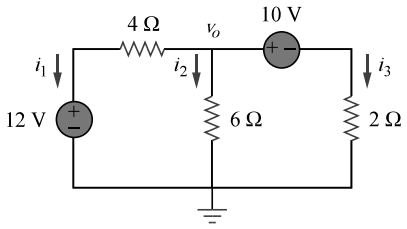


Figure 3.55 For Prob. 3.6.

3.7 Using nodal analysis, find v_o in the circuit of Fig. 3.56.

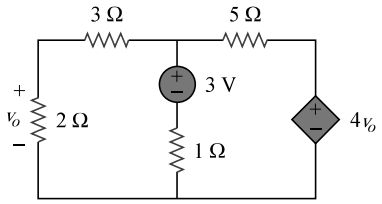


Figure 3.56 For Prob. 3.7.

3.8 Calculate v_o in the circuit in Fig. 3.57.

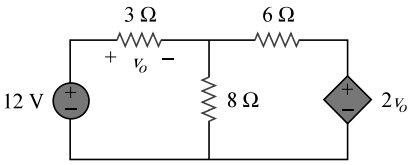


Figure 3.57 For Prob. 3.8.

3.9 Find i_o in the circuit in Fig. 3.58.

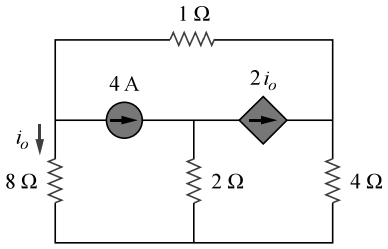


Figure 3.58 For Prob. 3.9.

3.10 Solve for i_1 and i_2 in the circuit in Fig. 3.22 (Section 3.5) using nodal analysis.

3.11 Use nodal analysis to find currents i_1 and i_2 in the circuit of Fig. 3.59.

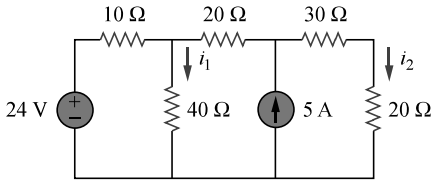


Figure 3.59 For Prob. 3.11.

3.12 Calculate v_1 and v_2 in the circuit in Fig. 3.60 using nodal analysis.

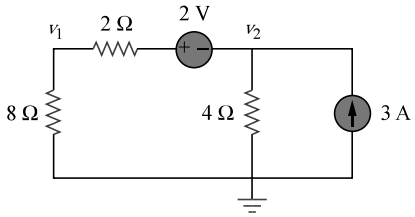


Figure 3.60 For Prob. 3.12.

3.13 Using nodal analysis, find v_o in the circuit of Fig. 3.61.

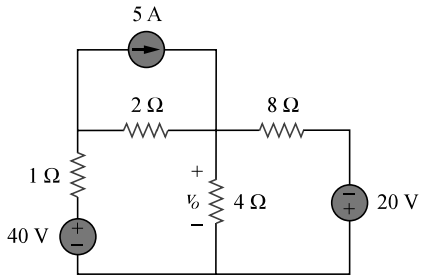


Figure 3.61 For Prob. 3.13.

3.14 Apply nodal analysis to find i_o and the power dissipated in each resistor in the circuit of Fig. 3.62.

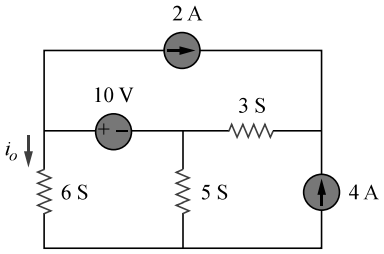


Figure 3.62 For Prob. 3.14.

3.15 Determine voltages v_1 through v_3 in the circuit of Fig. 3.63 using nodal analysis.

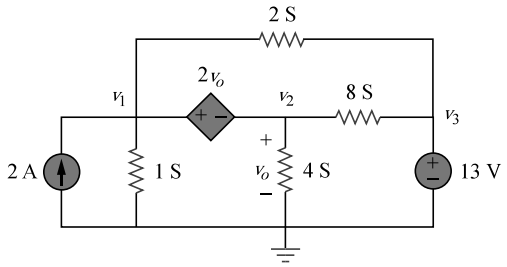


Figure 3.63 For Prob. 3.15.

3.16 Using nodal analysis, find current i_o in the circuit of Fig. 3.64.

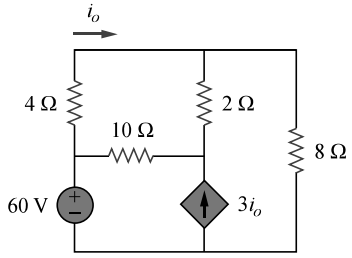


Figure 3.64 For Prob. 3.16.

3.17 Determine the node voltages in the circuit in Fig. 3.65 using nodal analysis.

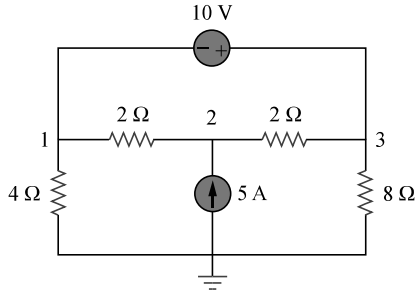


Figure 3.65 For Prob. 3.17.

3.18 For the circuit in Fig. 3.66, find v_1 and v_2 using nodal analysis.

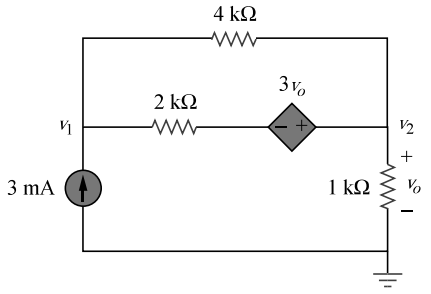


Figure 3.66 For Prob. 3.18.

3.19 Determine v_1 and v_2 in the circuit in Fig. 3.67.

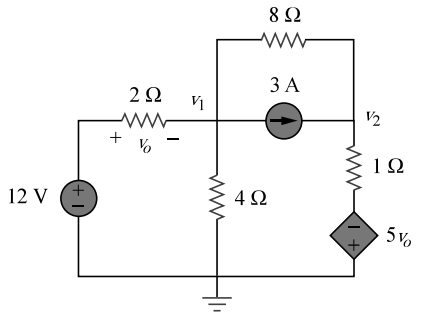


Figure 3.67 For Prob. 3.19.

3.20 Obtain v_1 and v_2 in the circuit of Fig. 3.68.

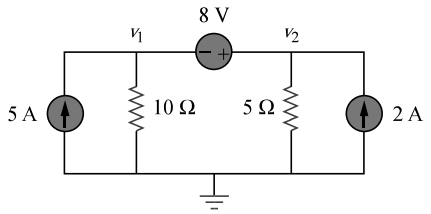


Figure 3.68 For Prob. 3.20.

3.21 Find v_o and i_o in the circuit in Fig. 3.69.

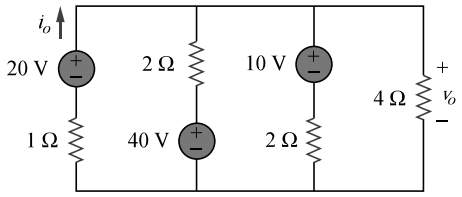


Figure 3.69 For Prob. 3.21.

***3.22** Use nodal analysis to determine voltages v_1, v_2, and v_3 in the circuit in Fig. 3.70.

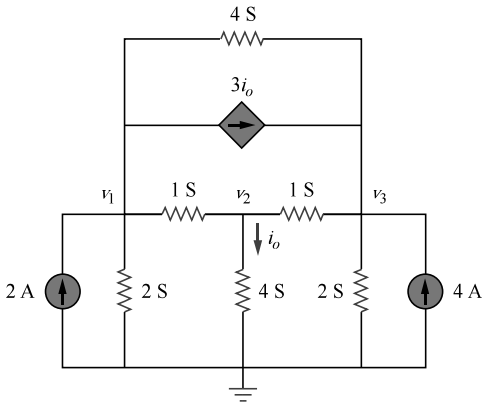


Figure 3.70 For Prob. 3.22.

3.23 Using nodal analysis, find v_o and i_o in the circuit of Fig. 3.71.

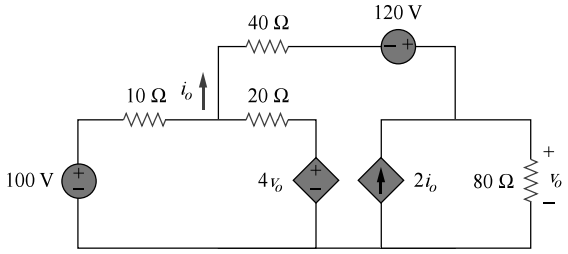


Figure 3.71 For Prob. 3.23.

3.24 Find the node voltages for the circuit in Fig. 3.72.

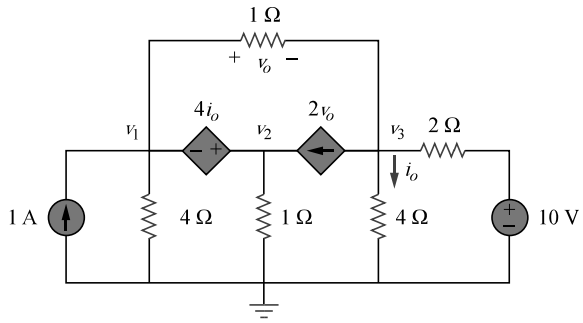


Figure 3.72 For Prob. 3.24.

*An asterisk indicates a challenging problem.

*3.25 Obtain the node voltages v_1, v_2, and v_3 in the circuit of Fig. 3.73.

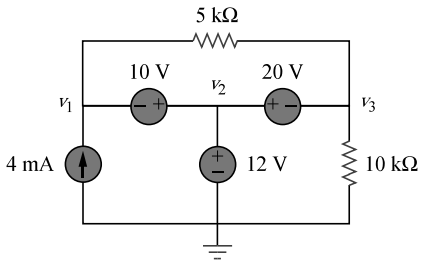
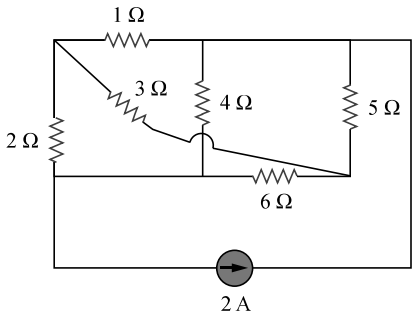


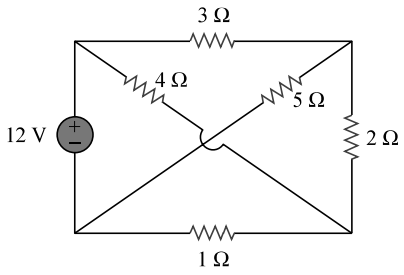
Figure 3.73 For Prob. 3.25.

Sections 3.4 and 3.5 Mesh Analysis

3.26 Which of the circuits in Fig. 3.74 is planar? For the planar circuit, redraw the circuits with no crossing branches.



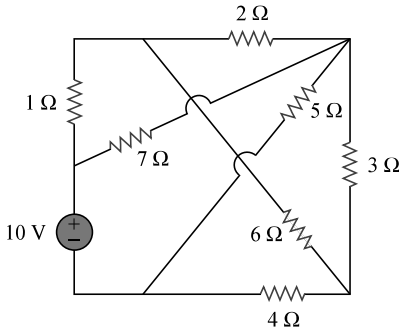
(a)



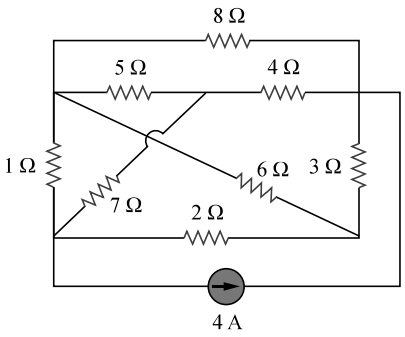
(b)

Figure 3.74 For Prob. 3.26.

3.27 Determine which of the circuits in Fig. 3.75 is planar and redraw it with no crossing branches.



(a)



(b)

Figure 3.75 For Prob. 3.27.

3.28 Rework Prob. 3.5 using mesh analysis.

3.29 Rework Prob. 3.6 using mesh analysis.

3.30 Solve Prob. 3.7 using mesh analysis.

3.31 Solve Prob. 3.8 using mesh analysis.

3.32 For the bridge network in Fig. 3.76, find i_o using mesh analysis.

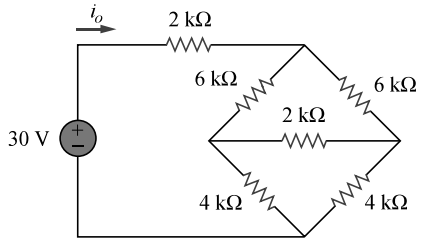


Figure 3.76 For Prob. 3.32.

3.33 Apply mesh analysis to find i in Fig. 3.77.

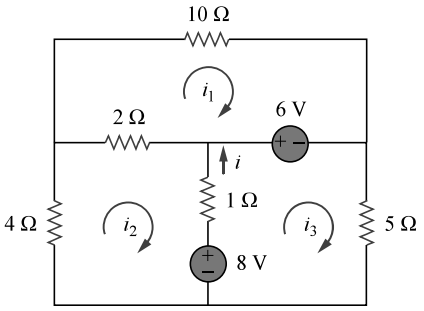


Figure 3.77 For Prob. 3.33.

3.34 Use mesh analysis to find v_{ab} and i_o in the circuit in Fig. 3.78.

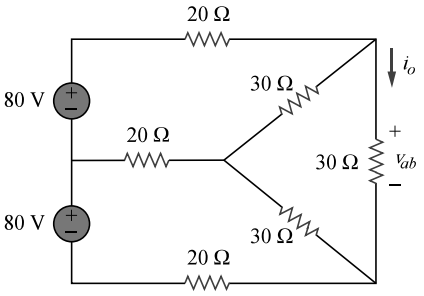


Figure 3.78 For Prob. 3.34.

3.35 Use mesh analysis to obtain i_o in the circuit of Fig. 3.79.

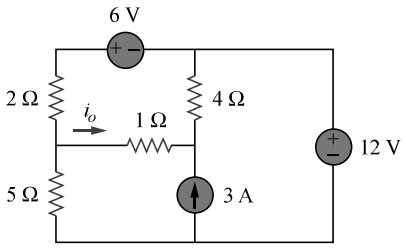


Figure 3.79 For Prob. 3.35.

3.36 Find current i in the circuit in Fig. 3.80.

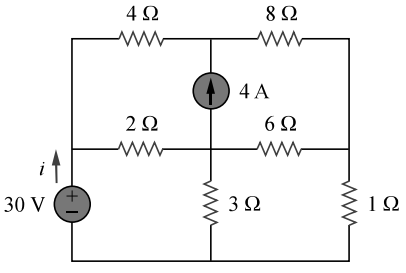


Figure 3.80 For Prob. 3.36.

3.37 Find v_o and i_o in the circuit of Fig. 3.81.

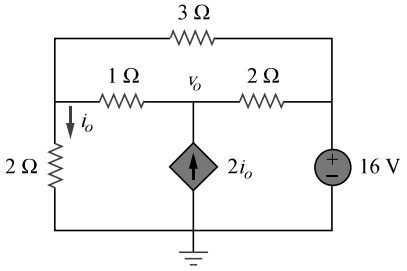


Figure 3.81 For Prob. 3.37.

3.38 Use mesh analysis to find the current i_o in the circuit in Fig. 3.82.

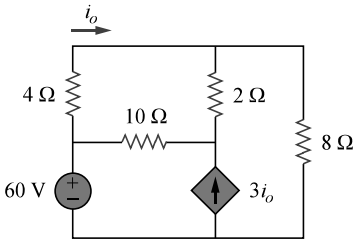


Figure 3.82 For Prob. 3.38.

3.39 Apply mesh analysis to find v_o in the circuit in Fig. 3.83.

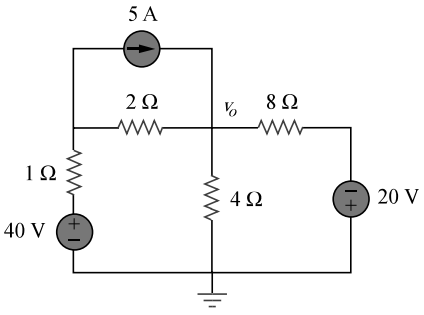


Figure 3.83 For Prob. 3.39.

3.40 Use mesh analysis to find i_1, i_2, and i_3 in the circuit of Fig. 3.84.

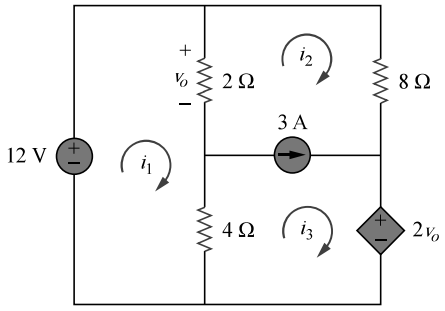


Figure 3.84 For Prob. 3.40.

3.41 Rework Prob. 3.11 using mesh analysis.

*__3.42__ In the circuit of Fig. 3.85, solve for i_1, i_2, and i_3.

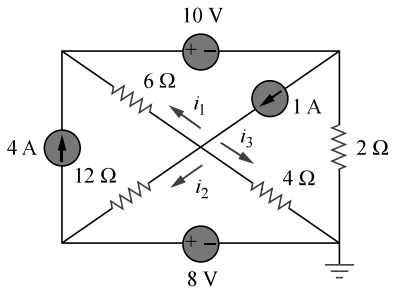


Figure 3.85 For Prob. 3.42.

3.43 Determine v_1 and v_2 in the circuit of Fig. 3.86.

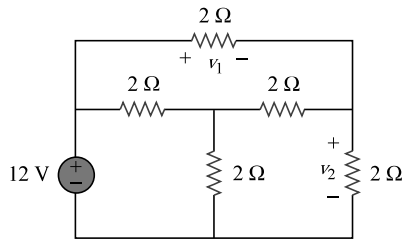


Figure 3.86 For Prob. 3.43.

3.44 Find i_1, i_2, and i_3 in the circuit in Fig. 3.87.

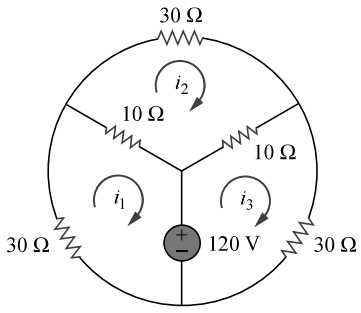


Figure 3.87 For Prob. 3.44.

3.45 Rework Prob. 3.23 using mesh analysis.

3.46 Calculate the power dissipated in each resistor in the circuit in Fig. 3.88.

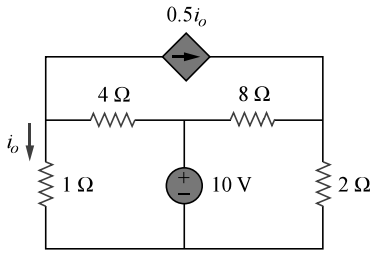


Figure 3.88 For Prob. 3.46.

3.47 Calculate the current gain i_o/i_s in the circuit of Fig. 3.89.

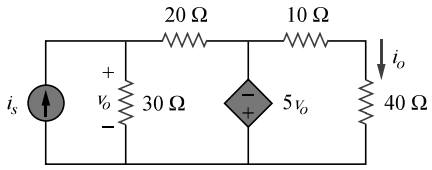


Figure 3.89 For Prob. 3.47.

3.48 Find the mesh currents i_1, i_2, and i_3 in the network of Fig. 3.90.

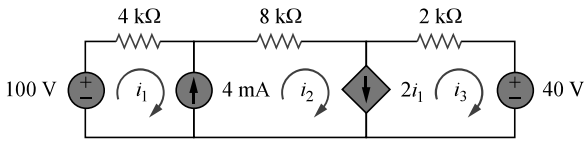


Figure 3.90 For Prob. 3.48.

3.49 Find v_x and i_x in the circuit shown in Fig. 3.91.

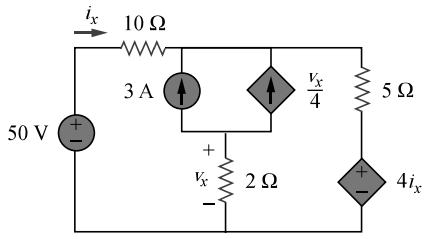


Figure 3.91 For Prob. 3.49.

4.1 INTRODUCTION

A major advantage of analyzing circuits using Kirchhoff's laws as we did in Chapter 3 is that we can analyze a circuit without tampering with its original configuration. A major disadvantage of this approach is that, for a large, complex circuit, tedious computation is involved.

The growth in areas of application of electric circuits has led to an evolution from simple to complex circuits. To handle the complexity, engineers over the years have developed some theorems to simplify circuit analysis. Such theorems include Thevenin's and Norton's theorems. Since these theorems are applicable to *linear* circuits, we first discuss the concept of circuit linearity. In addition to circuit theorems, we discuss the concepts of superposition, source transformation, and maximum power transfer in this chapter. The concepts we develop are applied in the last section to source modeling and resistance measurement.

4.2 LINEARITY PROPERTY

Linearity is the property of an element describing a linear relationship between cause and effect. Although the property applies to many circuit elements, we shall limit its applicability to resistors in this chapter. The property is a combination of both the homogeneity (scaling) property and the additivity property.

The homogeneity property requires that if the input (also called the *excitation*) is multiplied by a constant, then the output (also called the *response*) is multiplied by the same constant. For a resistor, for example, Ohm's law relates the input i to the output v,

$$v = iR \tag{4.1}$$

If the current is increased by a constant k, then the voltage increases correspondingly by k, that is,

$$\boxed{kiR = kv} \tag{4.2}$$

The additivity property requires that the response to a sum of inputs is the sum of the responses to each input applied separately. Using the voltage-current relationship of a resistor, if

$$v_1 = i_1 R \tag{4.3a}$$

and

$$v_2 = i_2 R \tag{4.3b}$$

then applying $(i_1 + i_2)$ gives

$$\boxed{v = (i_1 + i_2)R = i_1 R + i_2 R = v_1 + v_2} \tag{4.4}$$

We say that a resistor is a linear element because the voltage-current relationship satisfies both the homogeneity and the additivity properties.

In general, a circuit is linear if it is both additive and homogeneous. A linear circuit consists of only linear elements, linear dependent sources, and independent sources.

A linear circuit is one whose output is linearly related (or directly proportional) to its input.

Throughout this book we consider only linear circuits. Note that since $p = i^2R = v^2/R$ (making it a quadratic function rather than a linear one), the relationship between power and voltage (or current) is nonlinear. Therefore, the theorems covered in this chapter are not applicable to power.

To understand the linearity principle, consider the linear circuit shown in Fig. 4.1. The linear circuit has no independent sources inside it. It is excited by a voltage source v_s, which serves as the input. The circuit is terminated by a load R. We may take the current i through R as the output. Suppose $v_s = 10$ V gives $i = 2$ A. According to the linearity principle, $v_s = 1$ V will give $i = 0.2$ A. By the same token, $i = 1$ mA must be due to $v_s = 5$ mV.

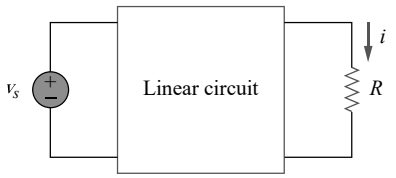


Figure 4.1 A linear circuit with input v_s and output i.

EXAMPLE 4.1

For the circuit in Fig. 4.2, find i_o when $v_s = 12$ V and $v_s = 24$ V.

Solution:

Applying KVL to the two loops, we obtain

$$12i_1 - 4i_2 + v_s = 0 \tag{4.1.1}$$

$$-4i_1 + 16i_2 - 3v_x - v_s = 0 \tag{4.1.2}$$

But $v_x = 2i_1$. Equation (4.1.2) becomes

$$-10i_1 + 16i_2 - v_s = 0 \tag{4.1.3}$$

Adding Eqs. (4.1.1) and (4.1.3) yields

$$2i_1 + 12i_2 = 0 \quad \Longrightarrow \quad i_1 = -6i_2$$

Substituting this in Eq. (4.1.1), we get

$$-76i_2 + v_s = 0 \quad \Longrightarrow \quad i_2 = \frac{v_s}{76}$$

When $v_s = 12$ V,

$$i_o = i_2 = \frac{12}{76} \text{ A}$$

When $v_s = 24$ V,

$$i_o = i_2 = \frac{24}{76} \text{ A}$$

showing that when the source value is doubled, i_o doubles.

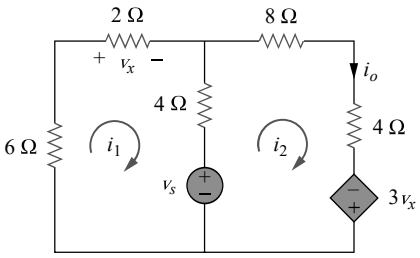


Figure 4.2 For Example 4.1.

PRACTICE PROBLEM 4.1

For the circuit in Fig. 4.3, find v_o when $i_s = 15$ and $i_s = 30$ A.

Answer: 10 V, 20 V.

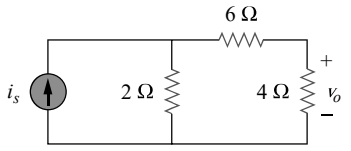


Figure 4.3 For Practice Prob. 4.1.

EXAMPLE 4.2

Assume $I_o = 1$ A and use linearity to find the actual value of I_o in the circuit in Fig. 4.4.

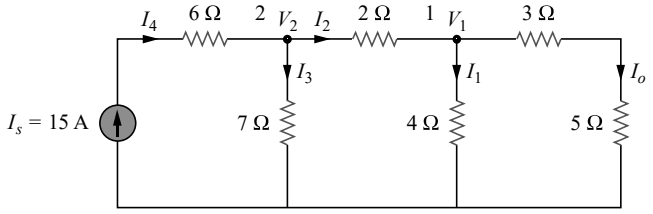


Figure 4.4 For Example 4.2.

Solution:

If $I_o = 1$ A, then $V_1 = (3+5)I_o = 8$ V and $I_1 = V_1/4 = 2$ A. Applying KCL at node 1 gives

$$I_2 = I_1 + I_o = 3 \text{ A}$$

$$V_2 = V_1 + 2I_2 = 8 + 6 = 14 \text{ V}, \qquad I_3 = \frac{V_2}{7} = 2 \text{ A}$$

Applying KCL at node 2 gives

$$I_4 = I_3 + I_2 = 5 \text{ A}$$

Therefore, $I_s = 5$ A. This shows that assuming $I_o = 1$ gives $I_s = 5$ A; the actual source current of 15 A will give $I_o = 3$ A as the actual value.

PRACTICE PROBLEM 4.2

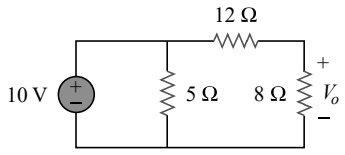


Figure 4.5 For Practice Prob. 4.2

Assume that $V_o = 1$ V and use linearity to calculate the actual value of V_o in the circuit of Fig. 4.5.

Answer: 4 V.

4.3 SUPERPOSITION

If a circuit has two or more independent sources, one way to determine the value of a specific variable (voltage or current) is to use nodal or mesh analysis as in Chapter 3. Another way is to determine the contribution of each independent source to the variable and then add them up. The latter approach is known as the *superposition.*

The idea of superposition rests on the linearity property.

Superposition is not limited to circuit analysis but is applicable in many fields where cause and effect bear a linear relationship to one another.

> The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately. However, to apply the superposition principle, we must keep two things in mind:

1. We consider one independent source at a time while all other independent sources are *turned off*. This implies that we replace every voltage source by 0 V (or a short circuit), and every current source by 0 A (or an open circuit). This way we obtain a simpler and more manageable circuit.
2. Dependent sources are left intact because they are controlled by circuit variables.

Other terms such as *killed*, *made inactive*, *deadened*, or *set equal to zero* are often used to convey the same idea.

With these in mind, we apply the superposition principle in three steps:

Steps to Apply Superposition Principle:

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Analyzing a circuit using superposition has one major disadvantage: it may very likely involve more work. If the circuit has three independent sources, we may have to analyze three simpler circuits each providing the contribution due to the respective individual source. However, superposition does help reduce a complex circuit to simpler circuits through replacement of voltage sources by short circuits and of current sources by open circuits.

Keep in mind that superposition is based on linearity. For this reason, it is not applicable to the effect on power due to each source, because the power absorbed by a resistor depends on the square of the voltage or current. If the power value is needed, the current through (or voltage across) the element must be calculated first using superposition.

For example, when current i_1 flows through resistor R, the power is $p_1 = Ri_1^2$, and when current i_2 flows through R, the power is $p_2 = Ri_2^2$. If current $i_1 + i_2$ flows through R, the power absorbed is $p_3 = R(i_1 + i_2)^2 = Ri_1^2 + Ri_2^2 + 2Ri_1i_2 \neq p_1 + p_2$. Thus, the power relation is nonlinear.

EXAMPLE 4.3

Use the superposition theorem to find v in the circuit in Fig. 4.6.

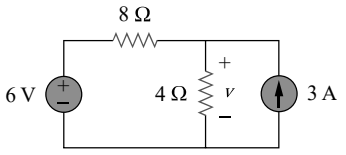


Figure 4.6 For Example 4.3.

Solution:

Since there are two sources, let

$$v = v_1 + v_2$$

where v_1 and v_2 are the contributions due to the 6-V voltage source and

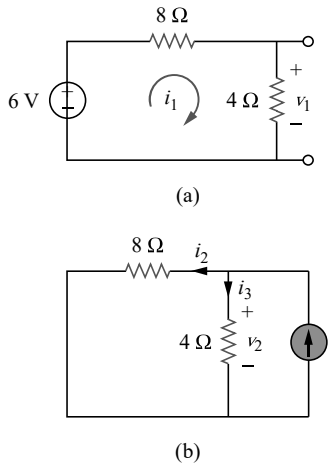


Figure 4.7 For Example 4.3: (a) calculating v_1, (b) calculating v_2.

the 3-A current source, respectively. To obtain v_1, we set the current source to zero, as shown in Fig. 4.7(a). Applying KVL to the loop in Fig. 4.7(a) gives

$$12i_1 - 6 = 0 \qquad \Longrightarrow \qquad i_1 = 0.5 \text{ A}$$

Thus,

$$v_1 = 4i_1 = 2 \text{ V}$$

We may also use voltage division to get v_1 by writing

$$v_1 = \frac{4}{4+8}(6) = 2 \text{ V}$$

To get v_2, we set the voltage source to zero, as in Fig. 4.7(b). Using current division,

$$i_3 = \frac{8}{4+8}(3) = 2 \text{ A}$$

Hence,

$$v_2 = 4i_3 = 8 \text{ V}$$

And we find

$$v = v_1 + v_2 = 2 + 8 = 10 \text{ V}$$

PRACTICE PROBLEM 4.3

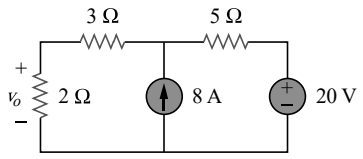


Figure 4.8 For Practice Prob. 4.3.

Using the superposition theorem, find v_o in the circuit in Fig. 4.8.

Answer: 12 V.

EXAMPLE 4.4

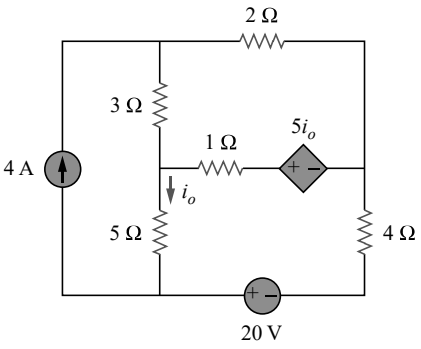


Figure 4.9 For Example 4.4.

Find i_o in the circuit in Fig. 4.9 using superposition.

Solution:

The circuit in Fig. 4.9 involves a dependent source, which must be left intact. We let

$$i_o = i_o' + i_o'' \tag{4.4.1}$$

where i_o' and i_o'' are due to the 4-A current source and 20-V voltage source respectively. To obtain i_o', we turn off the 20-V source so that we have the circuit in Fig. 4.10(a). We apply mesh analysis in order to obtain i_o'. For loop 1,

$$i_1 = 4 \text{ A} \tag{4.4.2}$$

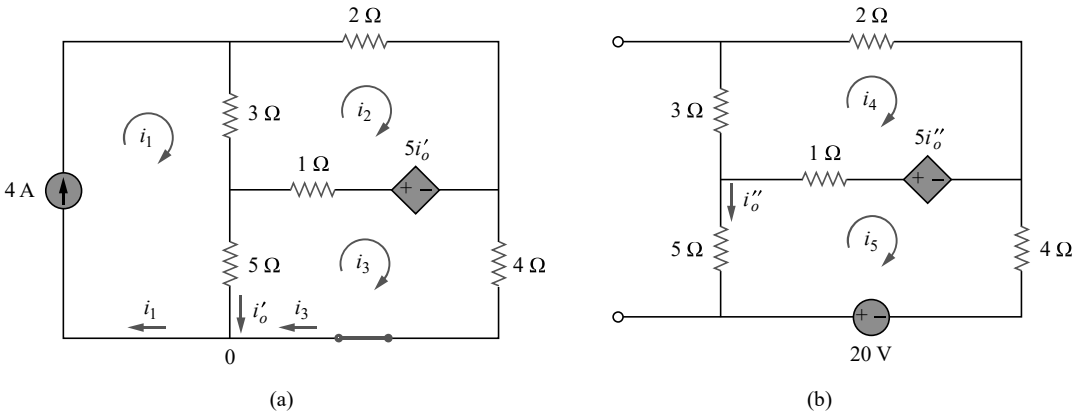


Figure 4.10 For Example 4.4: Applying superposition to (a) obtain i'_0, (b) obtain i''_0.

For loop 2,

$$-3i_1 + 6i_2 - 1i_3 - 5i'_o = 0 \tag{4.4.3}$$

For loop 3,

$$-5i_1 - 1i_2 + 10i_3 + 5i'_o = 0 \tag{4.4.4}$$

But at node 0,

$$i_3 = i_1 - i'_o = 4 - i'_o \tag{4.4.5}$$

Substituting Eqs. (4.4.2) and (4.4.5) into Eqs. (4.4.3) and (4.4.4) gives two simultaneous equations

$$3i_2 - 2i'_o = 8 \tag{4.4.6}$$

$$i_2 + 5i'_o = 20 \tag{4.4.7}$$

which can be solved to get

$$i'_o = \frac{52}{17}\ \text{A} \tag{4.4.8}$$

To obtain i''_o, we turn off the 4-A current source so that the circuit becomes that shown in Fig. 4.10(b). For loop 4, KVL gives

$$6i_4 - i_5 - 5i''_o = 0 \tag{4.4.9}$$

and for loop 5,

$$-i_4 + 10i_5 - 20 + 5i''_o = 0 \tag{4.4.10}$$

But $i_5 = -i''_o$. Substituting this in Eqs. (4.4.9) and (4.4.10) gives

$$6i_4 - 4i''_o = 0 \tag{4.4.11}$$

$$i_4 + 5i''_o = -20 \tag{4.4.12}$$

which we solve to get

$$i_o'' = -\frac{60}{17}\text{ A} \tag{4.4.13}$$

Now substituting Eqs. (4.4.8) and (4.4.13) into Eq. (4.4.1) gives

$$i_o = -\frac{8}{17} = -0.4706\text{ A}$$

PRACTICE PROBLEM 4.4

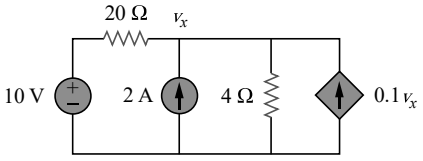


Figure 4.11 For Practice Prob. 4.4.

Use superposition to find v_x in the circuit in Fig. 4.11.

Answer: $v_x = 12.5$ V.

EXAMPLE 4.5

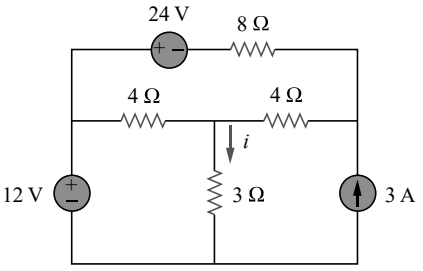


Figure 4.12 For Example 4.5.

For the circuit in Fig. 4.12, use the superposition theorem to find i.

Solution:

In this case, we have three sources. Let

$$i = i_1 + i_2 + i_3$$

where i_1, i_2, and i_3 are due to the 12-V, 24-V, and 3-A sources respectively. To get i_1, consider the circuit in Fig. 4.13(a). Combining 4 Ω (on the right-hand side) in series with 8 Ω gives 12 Ω. The 12 Ω in parallel with 4 Ω gives $12 \times 4/16 = 3$ Ω. Thus,

$$i_1 = \frac{12}{6} = 2\text{ A}$$

To get i_2, consider the circuit in Fig. 4.13(b). Applying mesh analysis,

$$16i_a - 4i_b + 24 = 0 \quad \Longrightarrow \quad 4i_a - i_b = -6 \tag{4.5.1}$$

$$7i_b - 4i_a = 0 \quad \Longrightarrow \quad i_a = \frac{7}{4}i_b \tag{4.5.2}$$

Substituting Eq. (4.5.2) into Eq. (4.5.1) gives

$$i_2 = i_b = -1$$

To get i_3, consider the circuit in Fig. 4.13(c). Using nodal analysis,

$$3 = \frac{v_2}{8} + \frac{v_2 - v_1}{4} \quad \Longrightarrow \quad 24 = 3v_2 - 2v_1 \tag{4.5.3}$$

$$\frac{v_2 - v_1}{4} = \frac{v_1}{4} + \frac{v_1}{3} \quad \Longrightarrow \quad v_2 = \frac{10}{3}v_1 \tag{4.5.4}$$

Substituting Eq. (4.5.4) into Eq. (4.5.3) leads to $v_1 = 3$ and

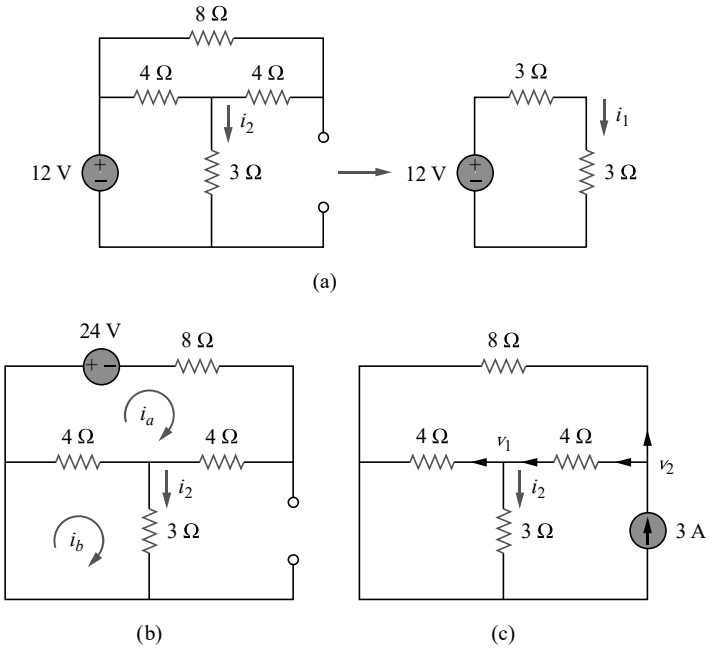


Figure 4.13 For Example 4.5.

$$i_3 = \frac{v_1}{3} = 1 \text{ A}$$

Thus,

$$i = i_1 + i_2 + i_3 = 2 - 1 + 1 = 2 \text{ A}$$

PRACTICE PROBLEM 4.5

Find i in the circuit in Fig. 4.14 using the superposition principle.

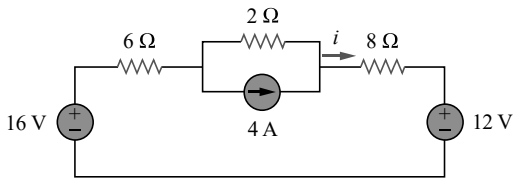


Figure 4.14 For Practice Prob. 4.5.

Answer: 0.75 A.

4.4 SOURCE TRANSFORMATION

We have noticed that series-parallel combination and wye-delta transformation help simplify circuits. *Source transformation* is another tool for simplifying circuits. Basic to these tools is the concept of *equivalence*.

We recall that an equivalent circuit is one whose v-i characteristics are identical with the original circuit.

In Section 3.6, we saw that node-voltage (or mesh-current) equations can be obtained by mere inspection of a circuit when the sources are all independent current (or all independent voltage) sources. It is therefore expedient in circuit analysis to be able to substitute a voltage source in series with a resistor for a current source in parallel with a resistor, or vice versa, as shown in Fig. 4.15. Either substitution is known as a *source transformation*.

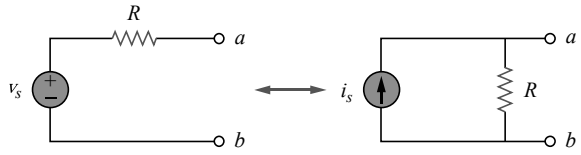


Figure 4.15 Transformation of independent sources.

A source transformation is the process of replacing a voltage source v_s in series with a resistor R by a current source i_s in parallel with a resistor R, or vice versa.

The two circuits in Fig. 4.15 are equivalent—provided they have the same voltage-current relation at terminals a-b. It is easy to show that they are indeed equivalent. If the sources are turned off, the equivalent resistance at terminals a-b in both circuits is R. Also, when terminals a-b are short-circuited, the short-circuit current flowing from a to b is $i_{sc} = v_s/R$ in the circuit on the left-hand side and $i_{sc} = i_s$ for the circuit on the right-hand side. Thus, $v_s/R = i_s$ in order for the two circuits to be equivalent. Hence, source transformation requires that

$$v_s = i_s R \qquad \text{or} \qquad i_s = \frac{v_s}{R} \tag{4.5}$$

Source transformation also applies to dependent sources, provided we carefully handle the dependent variable. As shown in Fig. 4.16, a dependent voltage source in series with a resistor can be transformed to a dependent current source in parallel with the resistor or vice versa.

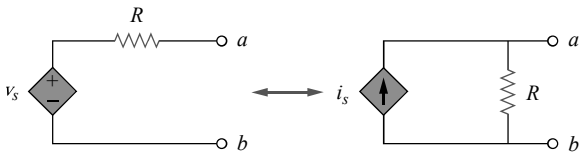


Figure 4.16 Transformation of dependent sources.

Like the wye-delta transformation we studied in Chapter 2, a source transformation does not affect the remaining part of the circuit. When

applicable, source transformation is a powerful tool that allows circuit manipulations to ease circuit analysis. However, we should keep the following points in mind when dealing with source transformation.

1. Note from Fig. 4.15 (or Fig. 4.16) that the arrow of the current source is directed toward the positive terminal of the voltage source.
2. Note from Eq. (4.5) that source transformation is not possible when $R = 0$, which is the case with an ideal voltage source. However, for a practical, nonideal voltage source, $R \neq 0$. Similarly, an ideal current source with $R = \infty$ cannot be replaced by a finite voltage source. More will be said on ideal and nonideal sources in Section 4.10.1.

EXAMPLE 4.6

Use source transformation to find v_o in the circuit in Fig. 4.17.

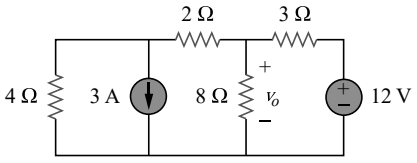


Figure 4.17 For Example 4.6.

Solution:

We first transform the current and voltage sources to obtain the circuit in Fig. 4.18(a). Combining the 4-Ω and 2-Ω resistors in series and transforming the 12-V voltage source gives us Fig. 4.18(b). We now combine the 3-Ω and 6-Ω resistors in parallel to get 2-Ω. We also combine the 2-A and 4-A current sources to get a 2-A source. Thus, by repeatedly applying source transformations, we obtain the circuit in Fig. 4.18(c).

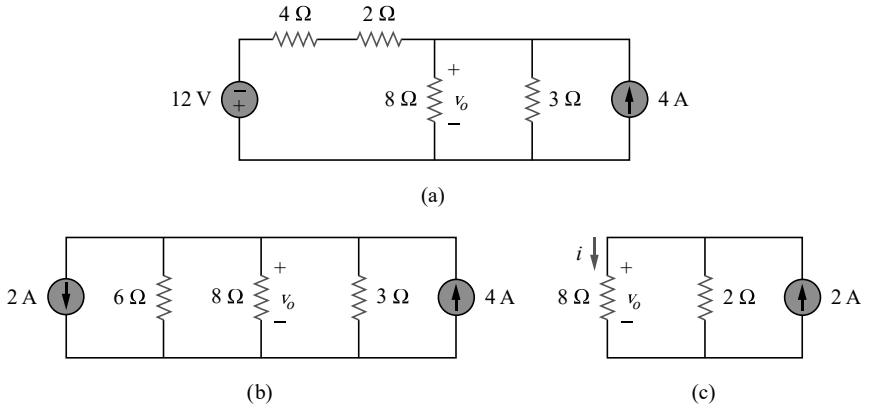


Figure 4.18 For Example 4.6.

We use current division in Fig. 4.18(c) to get

$$i = \frac{2}{2+8}(2) = 0.4$$

and

$$v_o = 8i = 8(0.4) = 3.2 \text{ V}$$

Alternatively, since the 8-Ω and 2-Ω resistors in Fig. 4.18(c) are in parallel, they have the same voltage v_o across them. Hence,

$$v_o = (8 \parallel 2)(2\text{ A}) = \frac{8 \times 2}{10}(2) = 3.2\text{ V}$$

PRACTICE PROBLEM 4.6

Find i_o in the circuit of Fig. 4.19 using source transformation.

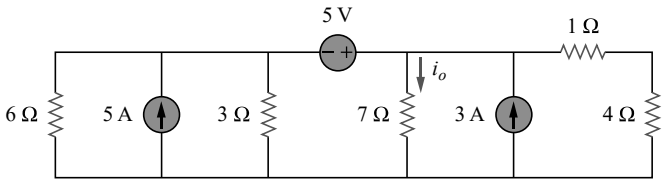


Figure 4.19 For Practice Prob. 4.6.

Answer: 1.78 A.

EXAMPLE 4.7

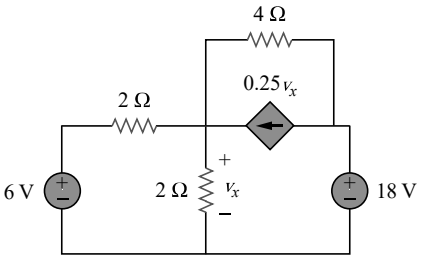


Figure 4.20 For Example 4.7.

Find v_x in Fig. 4.20 using source transformation.

Solution:

The circuit in Fig. 4.20 involves a voltage-controlled dependent current source. We transform this dependent current source as well as the 6-V independent voltage source as shown in Fig. 4.21(a). The 18-V voltage source is not transformed because it is not connected in series with any resistor. The two 2-Ω resistors in parallel combine to give a 1-Ω resistor, which is in parallel with the 3-A current source. The current is transformed to a voltage source as shown in Fig. 4.21(b). Notice that the terminals for v_x are intact. Applying KVL around the loop in Fig. 4.21(b) gives

$$-3 + 5i + v_x + 18 = 0 \tag{4.7.1}$$

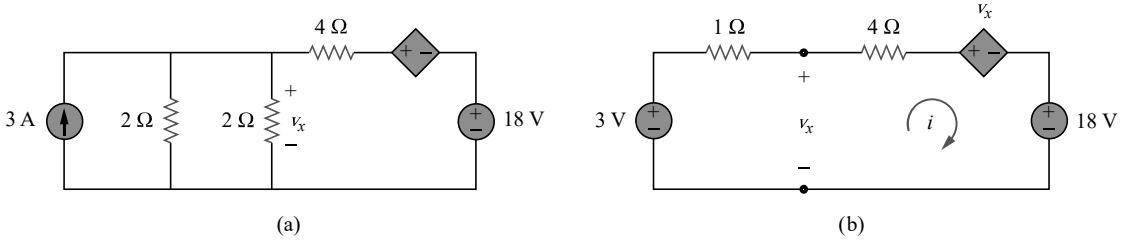


Figure 4.21 For Example 4.7: Applying source transformation to the circuit in Fig. 4.20.

Applying KVL to the loop containing only the 3-V voltage source, the 1-Ω resistor, and v_x yields

$$-3 + 1i + v_x = 0 \quad \Longrightarrow \quad v_x = 3 - i \tag{4.7.2}$$

Substituting this into Eq. (4.7.1), we obtain

$$15 + 5i + 3 - i = 0 \quad \Longrightarrow \quad i = -4.5 \text{ A}$$

Alternatively, we may apply KVL to the loop containing v_x, the 4-Ω resistor, the voltage-controlled dependent voltage source, and the 18-V voltage source in Fig. 4.21(b). We obtain

$$-v_x + 4i + v_x + 18 = 0 \quad \Longrightarrow \quad i = -4.5 \text{ A}$$

Thus, $v_x = 3 - i = 7.5$ V.

PRACTICE PROBLEM 4.7

Use source transformation to find i_x in the circuit shown in Fig. 4.22.

Answer: 1.176 A.

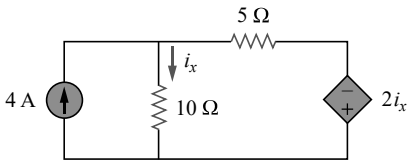


Figure 4.22 For Practice Prob. 4.7.

4.5 THEVENIN'S THEOREM

Electronic Testing Tutorials

It often occurs in practice that a particular element in a circuit is variable (usually called the *load*) while other elements are fixed. As a typical example, a household outlet terminal may be connected to different appliances constituting a variable load. Each time the variable element is changed, the entire circuit has to be analyzed all over again. To avoid this problem, Thevenin's theorem provides a technique by which the fixed part of the circuit is replaced by an equivalent circuit.

According to Thevenin's theorem, the linear circuit in Fig. 4.23(a) can be replaced by that in Fig. 4.23(b). (The load in Fig. 4.23 may be a single resistor or another circuit.) The circuit to the left of the terminals *a*-*b* in Fig. 4.23(b) is known as the *Thevenin equivalent circuit*; it was developed in 1883 by M. Leon Thevenin (1857–1926), a French telegraph engineer.

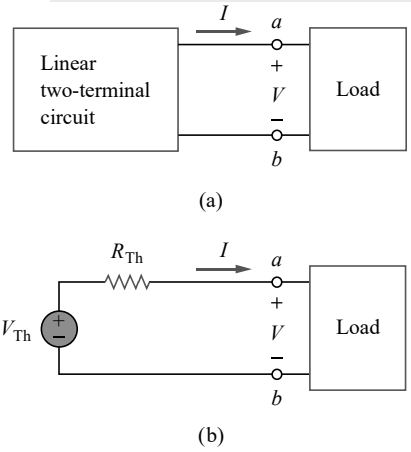


Figure 4.23 Replacing a linear two-terminal circuit by its Thevenin equivalent: (a) original circuit, (b) the Thevenin equivalent circuit.

> Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

The proof of the theorem will be given later, in Section 4.7. Our major concern right now is how to find the Thevenin equivalent voltage

V_{Th} and resistance R_{Th}. To do so, suppose the two circuits in Fig. 4.23 are equivalent. Two circuits are said to be *equivalent* if they have the same voltage-current relation at their terminals. Let us find out what will make the two circuits in Fig. 4.23 equivalent. If the terminals a-b are made open-circuited (by removing the load), no current flows, so that the open-circuit voltage across the terminals a-b in Fig. 4.23(a) must be equal to the voltage source V_{Th} in Fig. 4.23(b), since the two circuits are equivalent. Thus V_{Th} is the open-circuit voltage across the terminals as shown in Fig. 4.24(a); that is,

$$V_{\text{Th}} = v_{oc} \tag{4.6}$$

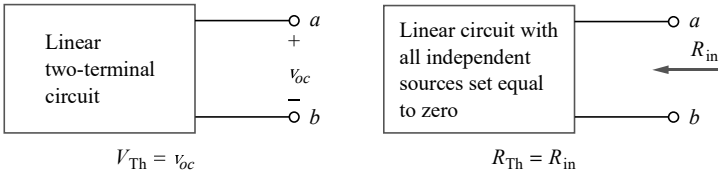


Figure 4.24 Finding V_{Th} and R_{Th}.

Again, with the load disconnected and terminals a-b open-circuited, we turn off all independent sources. The input resistance (or equivalent resistance) of the dead circuit at the terminals a-b in Fig. 4.23(a) must be equal to R_{Th} in Fig. 4.23(b) because the two circuits are equivalent. Thus, R_{Th} is the input resistance at the terminals when the independent sources are turned off, as shown in Fig. 4.24(b); that is,

$$R_{\text{Th}} = R_{\text{in}} \tag{4.7}$$

To apply this idea in finding the Thevenin resistance R_{Th}, we need to consider two cases.

CASE 1 If the network has no dependent sources, we turn off all independent sources. R_{Th} is the input resistance of the network looking between terminals a and b, as shown in Fig. 4.24(b).

CASE 2 If the network has dependent sources, we turn off all independent sources. As with superposition, dependent sources are not to be turned off because they are controlled by circuit variables. We apply a voltage source v_o at terminals a and b and determine the resulting current i_o. Then $R_{\text{Th}} = v_o/i_o$, as shown in Fig. 4.25(a). Alternatively, we may insert a current source i_o at terminals a-b as shown in Fig. 4.25(b) and find the terminal voltage v_o. Again $R_{\text{Th}} = v_o/i_o$. Either of the two approaches will give the same result. In either approach we may assume any value of v_o and i_o. For example, we may use $v_o = 1$ V or $i_o = 1$ A, or even use unspecified values of v_o or i_o.

It often occurs that R_{Th} takes a negative value. In this case, the negative resistance ($v = -iR$) implies that the circuit is supplying power.

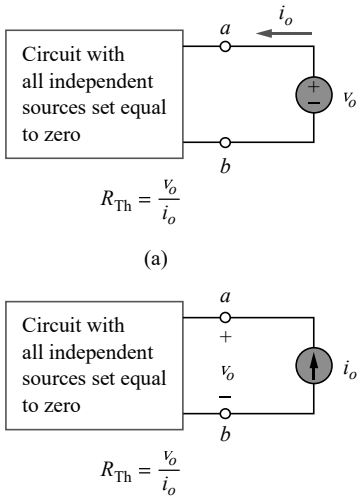


Figure 4.25 Finding R_{Th} when circuit has dependent sources.

Later we will see that an alternative way of finding R_{Th} is $R_{\text{Th}} = v_{oc}/i_{sc}$.

This is possible in a circuit with dependent sources; Example 4.10 will illustrate this.

Thevenin's theorem is very important in circuit analysis. It helps simplify a circuit. A large circuit may be replaced by a single independent voltage source and a single resistor. This replacement technique is a powerful tool in circuit design.

As mentioned earlier, a linear circuit with a variable load can be replaced by the Thevenin equivalent, exclusive of the load. The equivalent network behaves the same way externally as the original circuit. Consider a linear circuit terminated by a load R_L, as shown in Fig. 4.26(a). The current I_L through the load and the voltage V_L across the load are easily determined once the Thevenin equivalent of the circuit at the load's terminals is obtained, as shown in Fig. 4.26(b). From Fig. 4.26(b), we obtain

$$I_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L} \tag{4.8a}$$

$$V_L = R_L I_L = \frac{R_L}{R_{\text{Th}} + R_L} V_{\text{Th}} \tag{4.8b}$$

Note from Fig. 4.26(b) that the Thevenin equivalent is a simple voltage divider, yielding V_L by mere inspection.

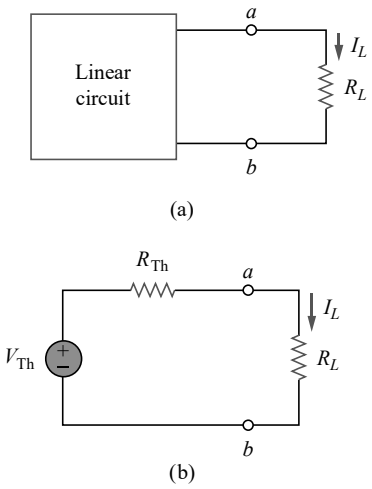


Figure 4.26 A circuit with a load: (a) original circuit, (b) Thevenin equivalent.

EXAMPLE 4.8

Find the Thevenin equivalent circuit of the circuit shown in Fig. 4.27, to the left of the terminals a-b. Then find the current through $R_L = 6$, 16, and 36 Ω.

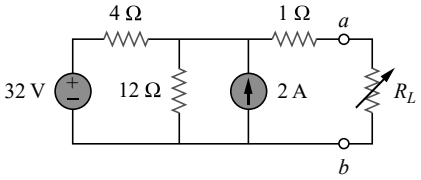


Figure 4.27 For Example 4.8.

Solution:

We find R_{Th} by turning off the 32-V voltage source (replacing it with a short circuit) and the 2-A current source (replacing it with an open circuit). The circuit becomes what is shown in Fig. 4.28(a). Thus,

$$R_{\text{Th}} = 4 \parallel 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\ \Omega$$

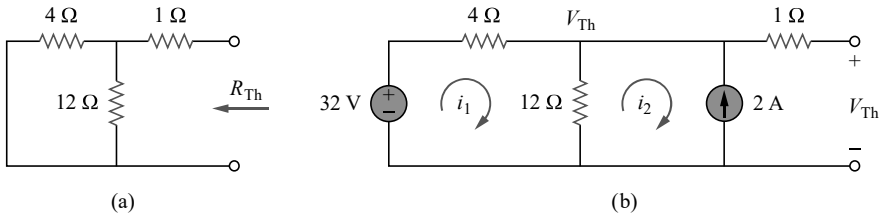


Figure 4.28 For Example 4.8: (a) finding R_{Th}, (b) finding V_{Th}.

To find V_{Th}, consider the circuit in Fig. 4.28(b). Applying mesh analysis to the two loops, we obtain

$$-32 + 4i_1 + 12(i_1 - i_2) = 0, \qquad i_2 = -2\ \text{A}$$

Solving for i_1, we get $i_1 = 0.5$ A. Thus,

$$V_{\text{Th}} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30 \text{ V}$$

Alternatively, it is even easier to use nodal analysis. We ignore the 1-Ω resistor since no current flows through it. At the top node, KCL gives

$$\frac{32 - V_{\text{Th}}}{4} + 2 = \frac{V_{\text{Th}}}{12}$$

or

$$96 - 3V_{\text{Th}} + 24 = V_{\text{Th}} \qquad \Longrightarrow \qquad V_{\text{Th}} = 30 \text{ V}$$

as obtained before. We could also use source transformation to find V_{Th}.

The Thevenin equivalent circuit is shown in Fig. 4.29. The current through R_L is

$$I_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L} = \frac{30}{4 + R_L}$$

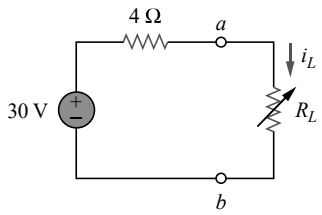


Figure 4.29 The Thevenin equivalent circuit for Example 4.8.

When $R_L = 6$,

$$I_L = \frac{30}{10} = 3 \text{ A}$$

When $R_L = 16$,

$$I_L = \frac{30}{20} = 1.5 \text{ A}$$

When $R_L = 36$,

$$I_L = \frac{30}{40} = 0.75 \text{ A}$$

PRACTICE PROBLEM 4.8

Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit in Fig. 4.30. Then find i.

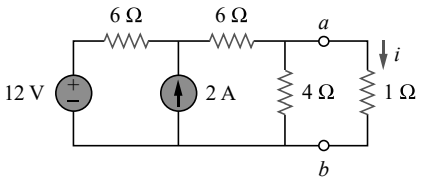


Figure 4.30 For Practice Prob. 4.8.

Answer: $V_{\text{Th}} = 6$ V, $R_{\text{Th}} = 3\ \Omega$, $i = 1.5$ A.

EXAMPLE 4.9

Find the Thevenin equivalent of the circuit in Fig. 4.31.

Solution:

This circuit contains a dependent source, unlike the circuit in the previous example. To find R_{Th}, we set the independent source equal to zero but leave the dependent source alone. Because of the presence of the dependent source, however, we excite the network with a voltage source v_o connected to the terminals as indicated in Fig. 4.32(a). We may set $v_o = 1$ V to ease calculation, since the circuit is linear. Our goal is to find the current i_o through the terminals, and then obtain $R_{\text{Th}} = 1/i_o$. (Alternatively, we may insert a 1-A current source, find the corresponding voltage v_o, and obtain $R_{\text{Th}} = v_o/1$.)

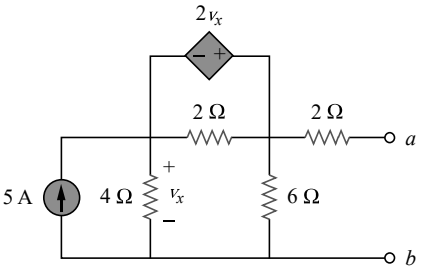


Figure 4.31 For Example 4.9.

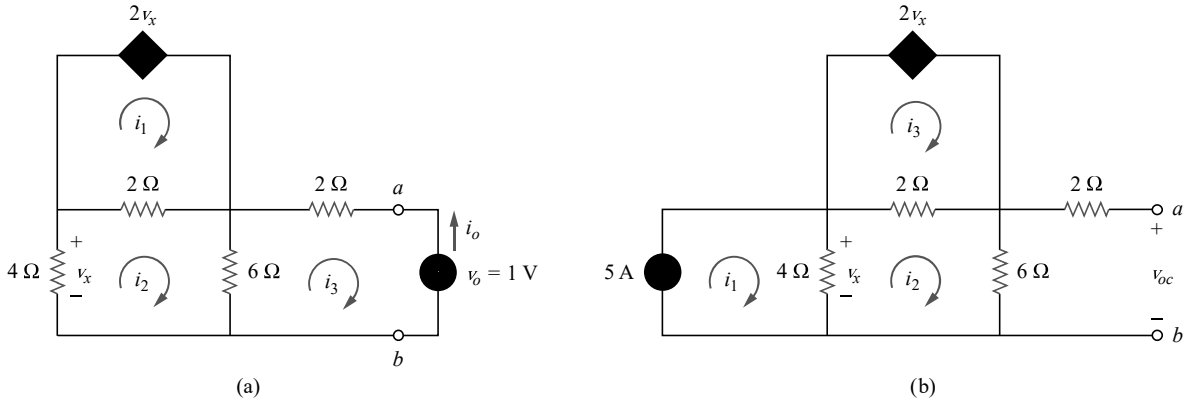


Figure 4.32 Finding R_{Th} and V_{Th} for Example 4.9.

Applying mesh analysis to loop 1 in the circuit in Fig. 4.32(a) results in

$$-2v_x + 2(i_1 - i_2) = 0 \qquad \text{or} \qquad v_x = i_1 - i_2$$

But $-4i_2 = v_x = i_1 - i_2$; hence,

$$i_1 = -3i_2 \tag{4.9.1}$$

For loops 2 and 3, applying KVL produces

$$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0 \tag{4.9.2}$$

$$6(i_3 - i_2) + 2i_3 + 1 = 0 \tag{4.9.3}$$

Solving these equations gives

$$i_3 = -\frac{1}{6}\text{ A}$$

But $i_o = -i_3 = 1/6$ A. Hence,

$$R_{\text{Th}} = \frac{1\text{ V}}{i_o} = 6\ \Omega$$

To get V_{Th}, we find v_{oc} in the circuit of Fig. 4.32(b). Applying mesh analysis, we get

$$i_1 = 5 \tag{4.9.4}$$

$$-2v_x + 2(i_3 - i_2) = 0 \quad \Longrightarrow \quad v_x = i_3 - i_2 \tag{4.9.5}$$

$$4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$$

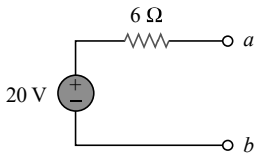


Figure 4.33 The Thevenin equivalent of the circuit in Fig. 4.31.

or

$$12i_2 - 4i_1 - 2i_3 = 0 \tag{4.9.6}$$

But $4(i_1 - i_2) = v_x$. Solving these equations leads to $i_2 = 10/3$. Hence,

$$V_{\text{Th}} = v_{oc} = 6i_2 = 20 \text{ V}$$

The Thevenin equivalent is as shown in Fig. 4.33.

PRACTICE PROBLEM 4.9

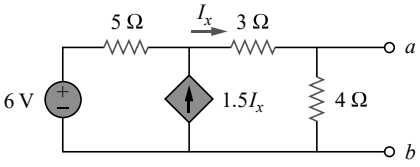


Figure 4.34 For Practice Prob. 4.9.

Find the Thevenin equivalent circuit of the circuit in Fig. 4.34 to the left of the terminals.

Answer: $V_{\text{Th}} = 5.33$ V, $R_{\text{Th}} = 0.44\ \Omega$.

EXAMPLE 4.10

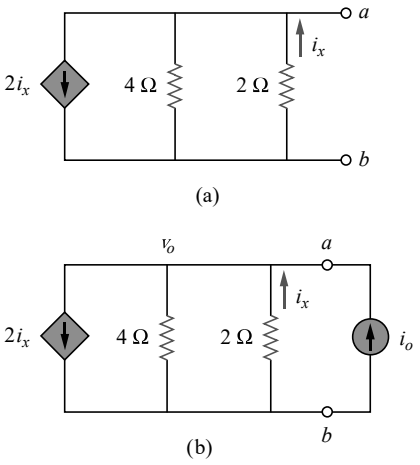


Figure 4.35 For Example 4.10.

Determine the Thevenin equivalent of the circuit in Fig. 4.35(a).

Solution:

Since the circuit in Fig. 4.35(a) has no independent sources, $V_{\text{Th}} = 0$ V. To find R_{Th}, it is best to apply a current source i_o at the terminals as shown in Fig. 4.35(b). Applying nodal analysis gives

$$i_o + i_x = 2i_x + \frac{v_o}{4} \tag{4.10.1}$$

But

$$i_x = \frac{0 - v_o}{2} = -\frac{v_o}{2} \tag{4.10.2}$$

Substituting Eq. (4.10.2) into Eq. (4.10.1) yields

$$i_o = i_x + \frac{v_o}{4} = -\frac{v_o}{2} + \frac{v_o}{4} = -\frac{v_o}{4} \quad \text{or} \quad v_o = -4i_o$$

Thus,

$$R_{\text{Th}} = \frac{v_o}{i_o} = -4\ \Omega$$

The negative value of the resistance tells us that, according to the passive sign convention, the circuit in Fig. 4.35(a) is supplying power. Of course, the resistors in Fig. 4.35(a) cannot supply power (they absorb power); it

is the dependent source that supplies the power. This is an example of how a dependent source and resistors could be used to simulate negative resistance.

Network Analysis

PRACTICE PROBLEM 4.10

Obtain the Thevenin equivalent of the circuit in Fig. 4.36.

Answer: $V_{Th} = 0$ V, $R_{Th} = -7.5\ \Omega$.

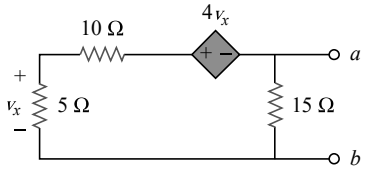


Figure 4.36 For Practice Prob. 4.10.

4.6 NORTON'S THEOREM

Electronic Testing Tutorials

In 1926, about 43 years after Thevenin published his theorem, E. L. Norton, an American engineer at Bell Telephone Laboratories, proposed a similar theorem.

> Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

Thus, the circuit in Fig. 4.37(a) can be replaced by the one in Fig. 4.37(b).

The proof of Norton's theorem will be given in the next section. For now, we are mainly concerned with how to get R_N and I_N. We find R_N in the same way we find R_{Th}. In fact, from what we know about source transformation, the Thevenin and Norton resistances are equal; that is,

$$R_N = R_{Th} \tag{4.9}$$

To find the Norton current I_N, we determine the short-circuit current flowing from terminal a to b in both circuits in Fig. 4.37. It is evident that the short-circuit current in Fig. 4.37(b) is I_N. This must be the same short-circuit current from terminal a to b in Fig. 4.37(a), since the two circuits are equivalent. Thus,

$$I_N = i_{sc} \tag{4.10}$$

shown in Fig. 4.38. Dependent and independent sources are treated the same way as in Thevenin's theorem.

Observe the close relationship between Norton's and Thevenin's theorems: $R_N = R_{Th}$ as in Eq. (4.9), and

$$I_N = \frac{V_{Th}}{R_{Th}} \tag{4.11}$$

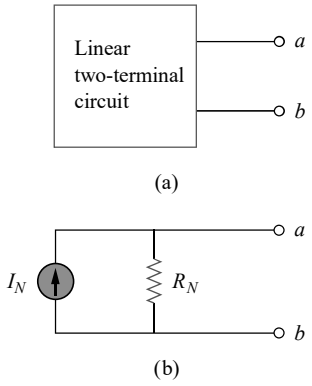


Figure 4.37 (a) Original circuit, (b) Norton equivalent circuit.

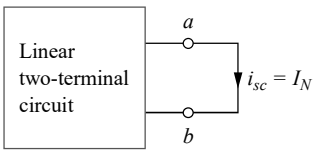


Figure 4.38 Finding Norton current I_N.

The Thevenin and Norton equivalent circuits are related by a source transformation.

This is essentially source transformation. For this reason, source transformation is often called Thevenin-Norton transformation.

Since V_{Th}, I_N, and R_{Th} are related according to Eq. (4.11), to determine the Thevenin or Norton equivalent circuit requires that we find:

- The open-circuit voltage v_{oc} across terminals a and b.
- The short-circuit current i_{sc} at terminals a and b.
- The equivalent or input resistance R_{in} at terminals a and b when all independent sources are turned off.

We can calculate any two of the three using the method that takes the least effort and use them to get the third using Ohm's law. Example 4.11 will illustrate this. Also, since

$$V_{\text{Th}} = v_{oc} \tag{4.12a}$$

$$I_N = i_{sc} \tag{4.12b}$$

$$R_{\text{Th}} = \frac{v_{oc}}{i_{sc}} = R_N \tag{4.12c}$$

the open-circuit and short-circuit tests are sufficient to find any Thevenin or Norton equivalent.

EXAMPLE 4.11

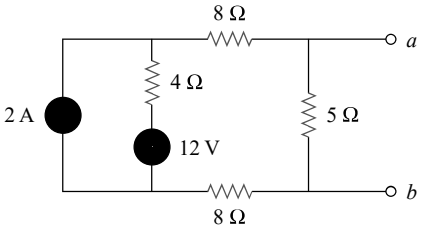


Figure 4.39 For Example 4.11.

Find the Norton equivalent circuit of the circuit in Fig. 4.39.

Solution:

We find R_N in the same way we find R_{Th} in the Thevenin equivalent circuit. Set the independent sources equal to zero. This leads to the circuit in Fig. 4.40(a), from which we find R_N. Thus,

$$R_N = 5 \parallel (8 + 4 + 8) = 5 \parallel 20 = \frac{20 \times 5}{25} = 4\ \Omega$$

To find I_N, we short-circuit terminals a and b, as shown in Fig. 4.40(b). We ignore the 5-Ω resistor because it has been short-circuited. Applying mesh analysis, we obtain

$$i_1 = 2\text{ A}, \qquad 20i_2 - 4i_1 - 12 = 0$$

From these equations, we obtain

$$i_2 = 1\text{ A} = i_{sc} = I_N$$

Alternatively, we may determine I_N from $V_{\text{Th}}/R_{\text{Th}}$. We obtain V_{Th} as the open-circuit voltage across terminals a and b in Fig. 4.40(c). Using mesh analysis, we obtain

$$i_3 = 2\text{ A}$$

$$25i_4 - 4i_3 - 12 = 0 \quad \Longrightarrow \quad i_4 = 0.8\text{ A}$$

and

$$v_{oc} = V_{\text{Th}} = 5i_4 = 4\text{ V}$$

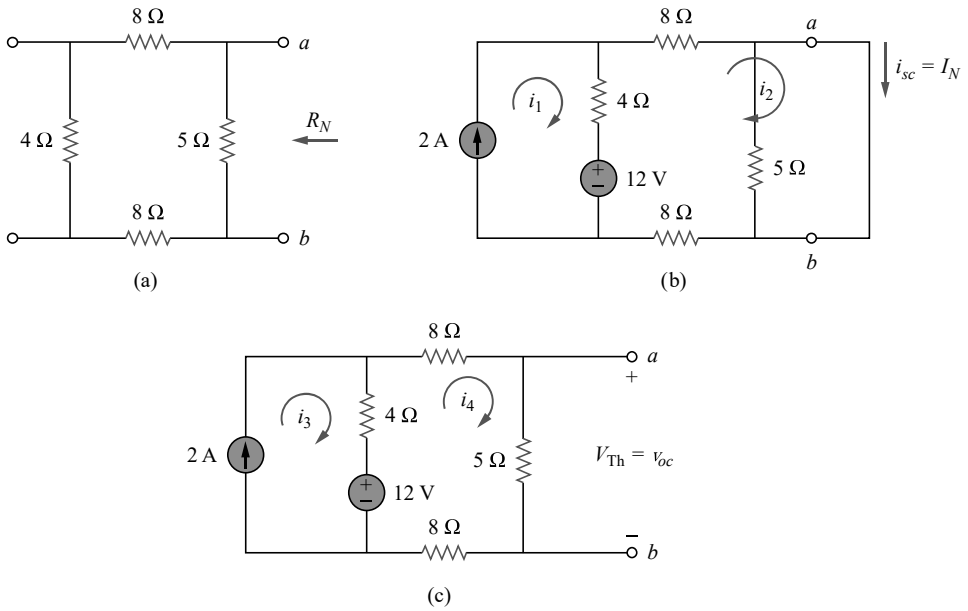


Figure 4.40 For Example 4.11; finding: (a) R_N, (b) $I_N = i_{sc}$, (c) $V_{\text{Th}} = v_{oc}$.

Hence,

$$I_N = \frac{V_{\text{Th}}}{R_{\text{Th}}} = \frac{4}{4} = 1 \text{ A}$$

as obtained previously. This also serves to confirm Eq. (4.7) that $R_{\text{Th}} = v_{oc}/i_{sc} = 4/1 = 4\ \Omega$. Thus, the Norton equivalent circuit is as shown in Fig. 4.41.

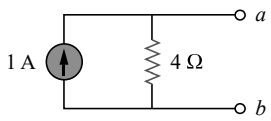


Figure 4.41 Norton equivalent of the circuit in Fig. 4.39.

PRACTICE PROBLEM 4.11

Find the Norton equivalent circuit for the circuit in Fig. 4.42.

Answer: $R_N = 3\ \Omega$, $I_N = 4.5$ A.

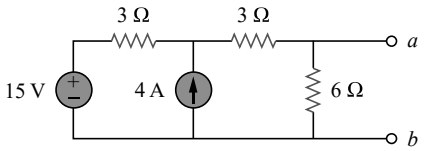


Figure 4.42 For Practice Prob. 4.11.

EXAMPLE 4.12

Using Norton's theorem, find R_N and I_N of the circuit in Fig. 4.43 at terminals *a*-*b*.

Solution:

To find R_N, we set the independent voltage source equal to zero and connect a voltage source of $v_o = 1$ V (or any unspecified voltage v_o) to the

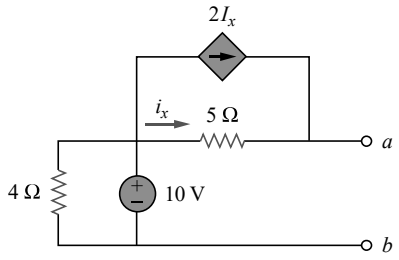


Figure 4.43 For Example 4.12.

terminals. We obtain the circuit in Fig. 4.44(a). We ignore the 4-Ω resistor because it is short-circuited. Also due to the short circuit, the 5-Ω resistor, the voltage source, and the dependent current source are all in parallel. Hence, $i_x = v_o/5 = 1/5 = 0.2$. At node a, $-i_o = i_x + 2i_x = 3i_x = 0.6$, and

$$R_N = \frac{v_o}{i_o} = \frac{1}{-0.6} = -1.67\ \Omega$$

To find I_N, we short-circuit terminals a and b and find the current i_{sc}, as indicated in Fig. 4.44(b). Note from this figure that the 4-Ω resistor, the 10-V voltage source, the 5-Ω resistor, and the dependent current source are all in parallel. Hence,

$$i_x = \frac{10 - 0}{5} = 2\ \text{A}$$

At node a, KCL gives

$$i_{sc} = i_x + 2i_x = 2 + 4 = 6\ \text{A}$$

Thus,

$$I_N = 6\ \text{A}$$

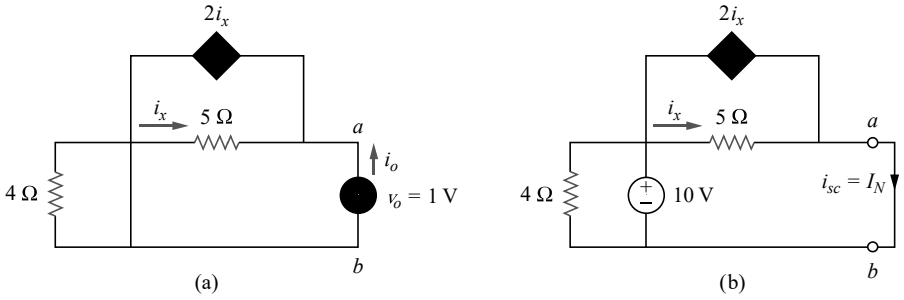


Figure 4.44 For Example 4.12: (a) finding R_N, (b) finding I_N.

PRACTICE PROBLEM 4.12

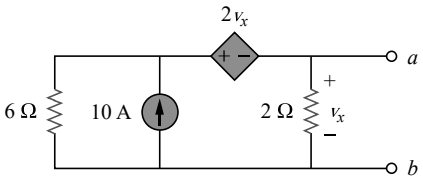


Figure 4.45 For Practice Prob. 4.12.

Find the Norton equivalent circuit of the circuit in Fig. 4.45.

Answer: $R_N = 1\ \Omega$, $I_N = 10\ \text{A}$.

†4.7 DERIVATIONS OF THEVENIN'S AND NORTON'S THEOREMS

In this section, we will prove Thevenin's and Norton's theorems using the superposition principle.

Consider the linear circuit in Fig. 4.46(a). It is assumed that the circuit contains resistors, and dependent and independent sources. We have access to the circuit via terminals a and b, through which current from an external source is applied. Our objective is to ensure that the voltage-current relation at terminals a and b is identical to that of the Thevenin equivalent in Fig. 4.46(b). For the sake of simplicity, suppose the linear circuit in Fig. 4.46(a) contains two independent voltage sources v_{s1} and v_{s2} and two independent current sources i_{s1} and i_{s2}. We may obtain any circuit variable, such as the terminal voltage v, by applying superposition. That is, we consider the contribution due to each independent source including the external source i. By superposition, the terminal voltage v is

$$v = A_0 i + A_1 v_{s1} + A_2 v_{s2} + A_3 i_{s1} + A_4 i_{s2} \tag{4.13}$$

where A_0, A_1, A_2, A_3, and A_4 are constants. Each term on the right-hand side of Eq. (4.13) is the contribution of the related independent source; that is, $A_0 i$ is the contribution to v due to the external current source i, $A_1 v_{s1}$ is the contribution due to the voltage source v_{s1}, and so on. We may collect terms for the internal independent sources together as B_0, so that Eq. (4.13) becomes

$$v = A_0 i + B_0 \tag{4.14}$$

where $B_0 = A_1 v_{s1} + A_2 v_{s2} + A_3 i_{s1} + A_4 i_{s2}$. We now want to evaluate the values of constants A_0 and B_0. When the terminals a and b are open-circuited, $i = 0$ and $v = B_0$. Thus B_0 is the open-circuit voltage v_{oc}, which is the same as V_{Th}, so

$$B_0 = V_{\text{Th}} \tag{4.15}$$

When all the internal sources are turned off, $B_0 = 0$. The circuit can then be replaced by an equivalent resistance R_{eq}, which is the same as R_{Th}, and Eq. (4.14) becomes

$$v = A_0 i = R_{\text{Th}} i \qquad \Longrightarrow \qquad A_0 = R_{\text{Th}} \tag{4.16}$$

Substituting the values of A_0 and B_0 in Eq. (4.14) gives

$$v = R_{\text{Th}} i + V_{\text{Th}} \tag{4.17}$$

which expresses the voltage-current relation at terminals a and b of the circuit in Fig. 4.46(b). Thus, the two circuits in Fig. 4.46(a) and 4.46(b) are equivalent.

When the same linear circuit is driven by a voltage source v as shown in Fig. 4.47(a), the current flowing into the circuit can be obtained by superposition as

$$i = C_0 v + D_0 \tag{4.18}$$

where $C_0 v$ is the contribution to i due to the external voltage source v and D_0 contains the contributions to i due to all internal independent sources. When the terminals a-b are short-circuited, $v = 0$ so that $i = D_0 = -i_{sc}$, where i_{sc} is the short-circuit current flowing out of terminal a, which is the same as the Norton current I_N, i.e.,

$$D_0 = -I_N \tag{4.19}$$

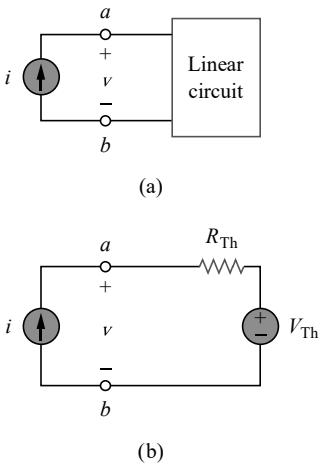


Figure 4.46 Derivation of Thevenin equivalent: (a) a current-driven circuit, (b) its Thevenin equivalent.

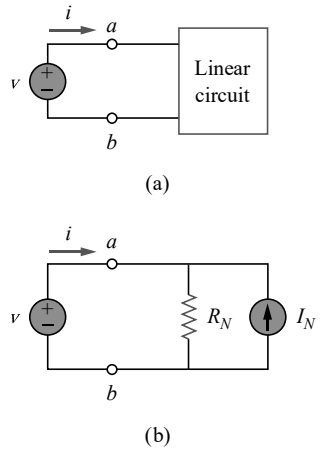


Figure 4.47 Derivation of Norton equivalent: (a) a voltage-driven circuit, (b) its Norton equivalent.

When all the internal independent sources are turned off, $D_0 = 0$ and the circuit can be replaced by an equivalent resistance R_{eq} (or an equivalent conductance $G_{eq} = 1/R_{eq}$), which is the same as R_{Th} or R_N. Thus Eq. (4.19) becomes

$$i = \frac{v}{R_{Th}} - I_N \tag{4.20}$$

This expresses the voltage-current relation at terminals a-b of the circuit in Fig. 4.47(b), confirming that the two circuits in Fig. 4.47(a) and 4.47(b) are equivalent.

4.8 MAXIMUM POWER TRANSFER

In many practical situations, a circuit is designed to provide power to a load. While for electric utilities, minimizing power losses in the process of transmission and distribution is critical for efficiency and economic reasons, there are other applications in areas such as communications where it is desirable to maximize the power delivered to a load. We now address the problem of delivering the maximum power to a load when given a system with known internal losses. It should be noted that this will result in significant internal losses greater than or equal to the power delivered to the load.

The Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load. We assume that we can adjust the load resistance R_L. If the entire circuit is replaced by its Thevenin equivalent except for the load, as shown in Fig. 4.48, the power delivered to the load is

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L \tag{4.21}$$

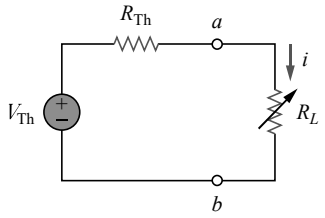


Figure 4.48 The circuit used for maximum power transfer.

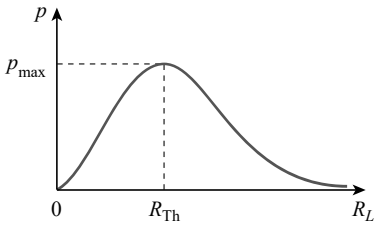


Figure 4.49 Power delivered to the load as a function of R_L.

For a given circuit, V_{Th} and R_{Th} are fixed. By varying the load resistance R_L, the power delivered to the load varies as sketched in Fig. 4.49. We notice from Fig. 4.49 that the power is small for small or large values of R_L but maximum for some value of R_L between 0 and ∞. We now want to show that this maximum power occurs when R_L is equal to R_{Th}. This is known as the *maximum power theorem.*

> Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$).

To prove the maximum power transfer theorem, we differentiate p in Eq. (4.21) with respect to R_L and set the result equal to zero. We obtain

$$\frac{dp}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th} + R_L)^2 - 2R_L(R_{Th} + R_L)}{(R_{Th} + R_L)^4}\right]$$

$$= V_{Th}^2 \left[\frac{(R_{Th} + R_L - 2R_L)}{(R_{Th} + R_L)^3}\right] = 0$$

This implies that

$$0 = (R_{\text{Th}} + R_L - 2R_L) = (R_{\text{Th}} - R_L) \tag{4.22}$$

which yields

$$\boxed{R_L = R_{\text{Th}}} \tag{4.23}$$

showing that the maximum power transfer takes place when the load resistance R_L equals the Thevenin resistance R_{Th}. We can readily confirm that Eq. (4.23) gives the maximum power by showing that $d^2p/dR_L^2 < 0$.

The source and load are said to be *matched* when $R_L = R_{\text{Th}}$.

The maximum power transferred is obtained by substituting Eq. (4.23) into Eq. (4.21), for

$$\boxed{p_{\max} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}}} \tag{4.24}$$

Equation (4.24) applies only when $R_L = R_{\text{Th}}$. When $R_L \neq R_{\text{Th}}$, we compute the power delivered to the load using Eq. (4.21).

EXAMPLE 4.13

Find the value of R_L for maximum power transfer in the circuit of Fig. 4.50. Find the maximum power.

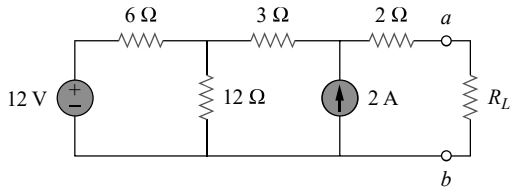


Figure 4.50 For Example 4.13.

Solution:

We need to find the Thevenin resistance R_{Th} and the Thevenin voltage V_{Th} across the terminals a-b. To get R_{Th}, we use the circuit in Fig. 4.51(a) and obtain

$$R_{\text{Th}} = 2 + 3 + 6 \parallel 12 = 5 + \frac{6 \times 12}{18} = 9\ \Omega$$

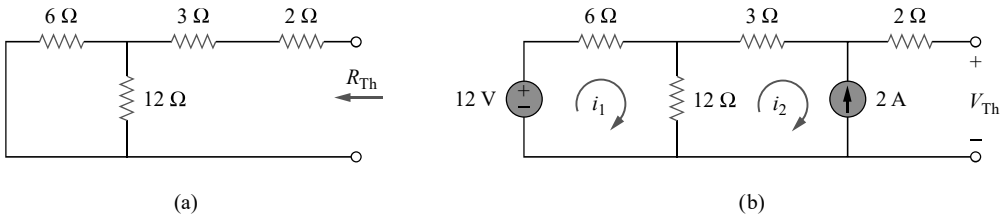


Figure 4.51 For Example 4.13: (a) finding R_{Th}, (b) finding V_{Th}.

To get V_{Th}, we consider the circuit in Fig. 4.51(b). Applying mesh analysis,

$$-12 + 18i_1 - 12i_2 = 0, \qquad i_2 = -2 \text{ A}$$

Solving for i_1, we get $i_1 = -2/3$. Applying KVL around the outer loop to get V_{Th} across terminals a-b, we obtain

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{\text{Th}} = 0 \qquad \Longrightarrow \qquad V_{\text{Th}} = 22 \text{ V}$$

For maximum power transfer,

$$R_L = R_{\text{Th}} = 9 \ \Omega$$

and the maximum power is

$$p_{\max} = \frac{V_{\text{Th}}^2}{4R_L} = \frac{22^2}{4 \times 9} = 13.44 \text{ W}$$

PRACTICE PROBLEM 4.13

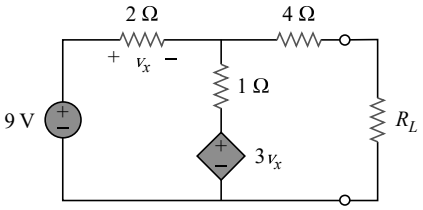


Figure 4.52 For Practice Prob. 4.13.

Determine the value of R_L that will draw the maximum power from the rest of the circuit in Fig. 4.52. Calculate the maximum power.

Answer: 4.22 Ω, 2.901 W.

4.9 VERIFYING CIRCUIT THEOREMS WITH *PSPICE*

In this section, we learn how to use *PSpice* to verify the theorems covered in this chapter. Specifically, we will consider using dc sweep analysis to find the Thevenin or Norton equivalent at any pair of nodes in a circuit and the maximum power transfer to a load. The reader is advised to read Section D.3 of Appendix D in preparation for this section.

To find the Thevenin equivalent of a circuit at a pair of open terminals using *PSpice*, we use the schematic editor to draw the circuit and insert an independent probing current source, say, Ip, at the terminals. The probing current source must have a part name ISRC. We then perform a DC Sweep on Ip, as discussed in Section D.3. Typically, we may let the current through Ip vary from 0 to 1 A in 0.1-A increments. After simulating the circuit, we use Probe to display a plot of the voltage across Ip versus the current through Ip. The zero intercept of the plot gives us the Thevenin equivalent voltage, while the slope of the plot is equal to the Thevenin resistance.

To find the Norton equivalent involves similar steps except that we insert a probing independent voltage source (with a part name VSRC), say, Vp, at the terminals. We perform a DC Sweep on Vp and let Vp vary from 0 to 1 V in 0.1-V increments. A plot of the current through

4.5 For the circuit in Fig. 4.73, assume $v_o = 1$ V, and use linearity to find the actual value of v_o.

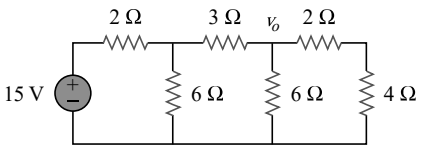


Figure 4.73 For Prob. 4.5.

Section 4.3 Superposition

4.6 Apply superposition to find i in the circuit of Fig. 4.74.

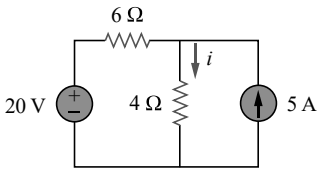


Figure 4.74 For Prob. 4.6.

4.7 Given the circuit in Fig. 4.75, calculate i_x and the power dissipated by the 10-Ω resistor using superposition.

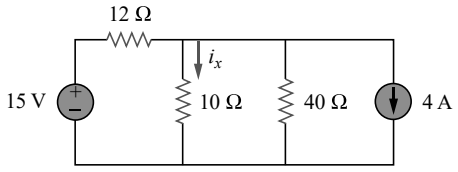


Figure 4.75 For Prob. 4.7.

4.8 For the circuit in Fig. 4.76, find the terminal voltage V_{ab} using superposition.

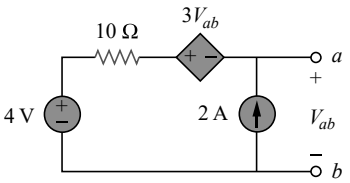


Figure 4.76 For Prob. 4.8.

4.9 Use superposition principle to find i in Fig. 4.77.

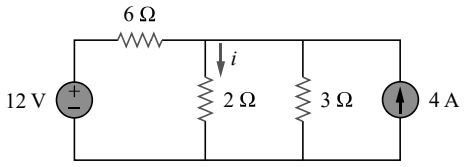


Figure 4.77 For Prob. 4.9.

4.10 Determine v_o in the circuit of Fig. 4.78 using the superposition principle.

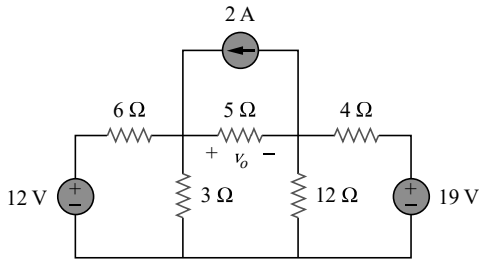


Figure 4.78 For Prob. 4.10.

4.11 Apply the superposition principle to find v_o in the circuit of Fig. 4.79.

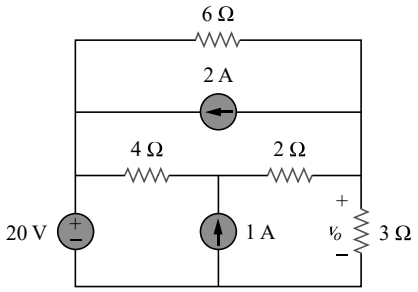


Figure 4.79 For Prob. 4.11.

4.12 For the circuit in Fig. 4.80, use superposition to find i. Calculate the power delivered to the 3-Ω resistor.

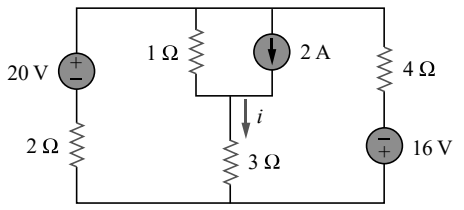


Figure 4.80 For Probs. 4.12 and 4.45.

4.13 Given the circuit in Fig. 4.81, use superposition to get i_o.

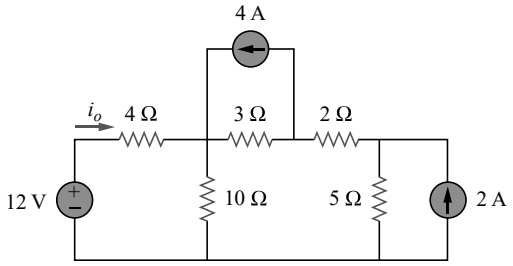


Figure 4.81 For Probs. 4.13 and 4.23.

4.14 Use superposition to obtain v_x in the circuit of Fig. 4.82. Check your result using *PSpice*.

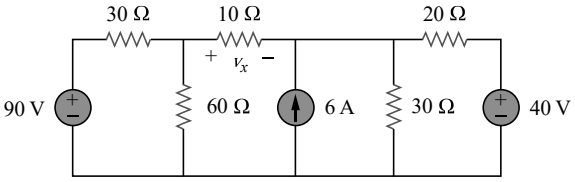


Figure 4.82 For Prob. 4.14.

4.15 Find v_x in Fig. 4.83 by superposition.

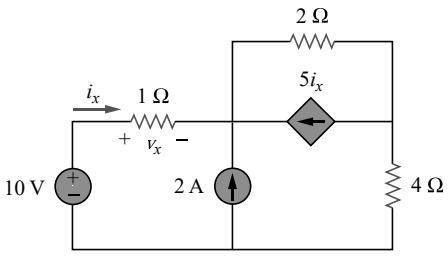


Figure 4.83 For Prob. 4.15.

4.16 Use superposition to solve for i_x in the circuit of Fig. 4.84.

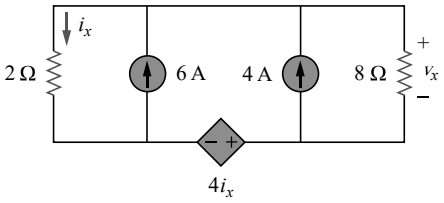


Figure 4.84 For Prob. 4.16.

Section 4.4 Source Transformation

4.17 Find i in Prob. 4.9 using source transformation.

4.18 Apply source transformation to determine v_o and i_o in the circuit in Fig. 4.85.

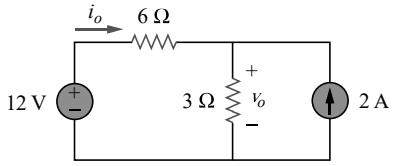


Figure 4.85 For Prob. 4.18.

4.19 For the circuit in Fig. 4.86, use source transformation to find i.

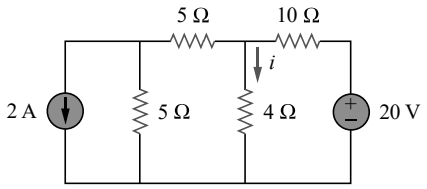


Figure 4.86 For Prob. 4.19.

4.20 Obtain v_o in the circuit of Fig. 4.87 using source transformation. Check your result using *PSpice*.

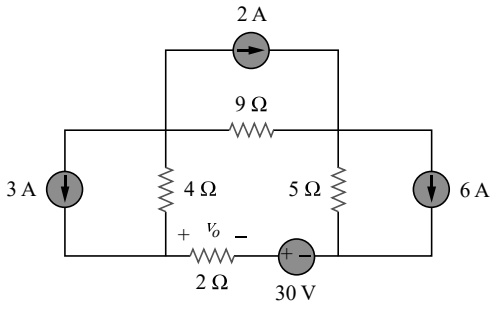


Figure 4.87 For Prob. 4.20.

4.21 Use source transformation to solve Prob. 4.14.

4.22 Apply source transformation to find v_x in the circuit of Fig. 4.88.

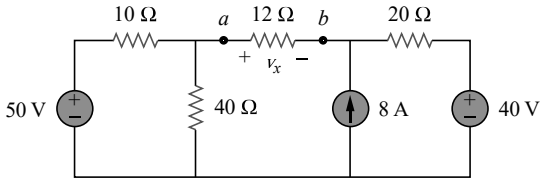


Figure 4.88 For Probs. 4.22 and 4.32.

4.23 Given the circuit in Fig. 4.81, use source transformation to find i_o.

4.24 Use source transformation to find v_o in the circuit of Fig. 4.89.

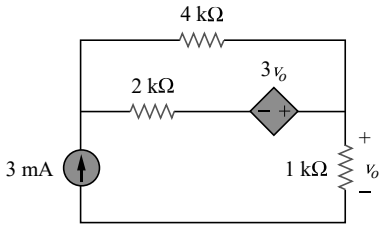


Figure 4.89 For Prob. 4.24.

4.25 Determine v_x in the circuit of Fig. 4.90 using source transformation.

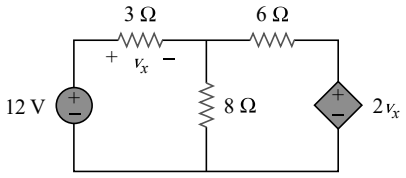


Figure 4.90 For Prob. 4.25.

4.26 Use source transformation to find i_x in the circuit of Fig. 4.91.

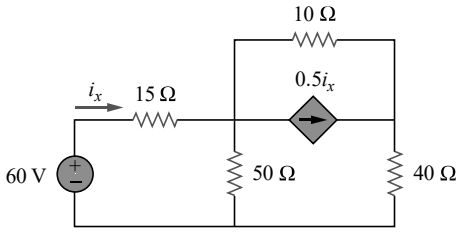
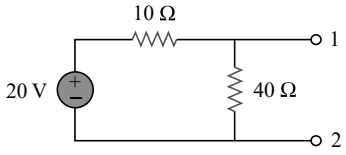


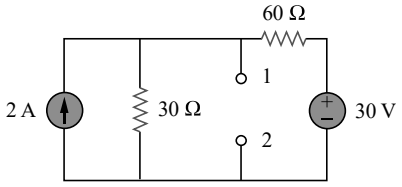
Figure 4.91 For Prob. 4.26.

Sections 4.5 and 4.6 Thevenin's and Norton's Theorems

4.27 Determine R_{Th} and V_{Th} at terminals 1-2 of each of the circuits in Fig. 4.92.



(a)



(b)

Figure 4.92 For Probs. 4.27 and 4.37.

4.28 Find the Thevenin equivalent at terminals a-b of the circuit in Fig. 4.93.

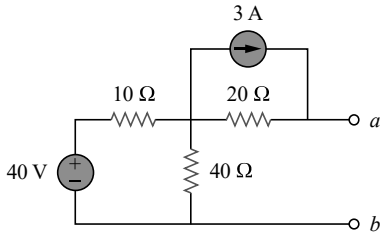


Figure 4.93 For Probs. 4.28 and 4.39.

4.29 Use Thevenin's theorem to find v_o in Prob. 4.10.

4.30 Solve for the current i in the circuit of Fig. 4.94 using Thevenin's theorem. (*Hint:* Find the Thevenin equivalent across the 12-Ω resistor.)

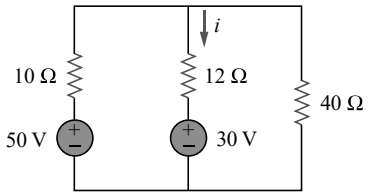


Figure 4.94 For Prob. 4.30.

4.31 For Prob. 4.8, obtain the Thevenin equivalent at terminals a-b.

4.32 Given the circuit in Fig. 4.88, obtain the Thevenin equivalent at terminals *a-b* and use the result to get v_x.

*__4.33__ For the circuit in Fig. 4.95, find the Thevenin equivalent between terminals *a* and *b*.

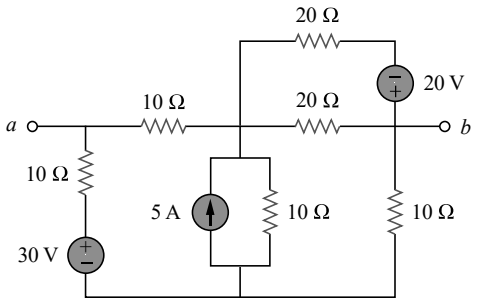


Figure 4.95 For Prob. 4.33.

4.34 Find the Thevenin equivalent looking into terminals *a-b* of the circuit in Fig. 4.96 and solve for i_x.

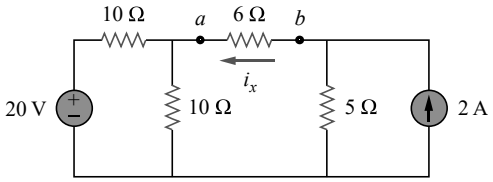


Figure 4.96 For Prob. 4.34.

4.35 For the circuit in Fig. 4.97, obtain the Thevenin equivalent as seen from terminals:
(a) *a-b* (b) *b-c*

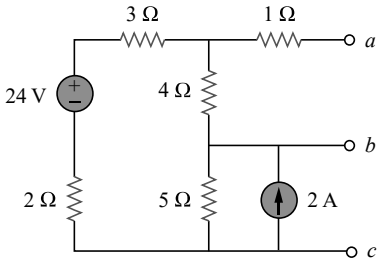


Figure 4.97 For Prob. 4.35.

4.36 Find the Norton equivalent of the circuit in Fig. 4.98.

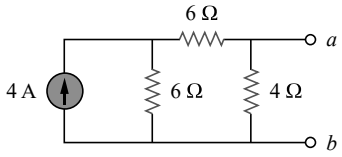


Figure 4.98 For Prob. 4.36.

4.37 Obtain R_N and I_N at terminals 1 and 2 of each of the circuits in Fig. 4.92.

4.38 Determine the Norton equivalent at terminals *a-b* for the circuit in Fig. 4.99.

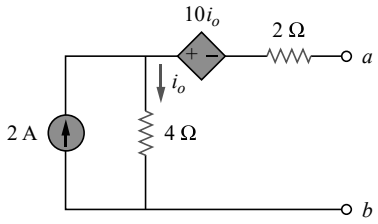


Figure 4.99 For Prob. 4.38.

4.39 Find the Norton equivalent looking into terminals *a-b* of the circuit in Fig. 4.93.

4.40 Obtain the Norton equivalent of the circuit in Fig. 4.100 to the left of terminals *a-b*. Use the result to find current *i*.

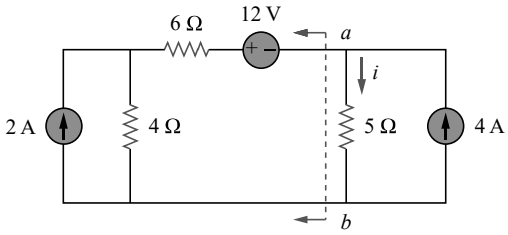


Figure 4.100 For Prob. 4.40.

4.41 Given the circuit in Fig. 4.101, obtain the Norton equivalent as viewed from terminals:
(a) *a-b* (b) *c-d*

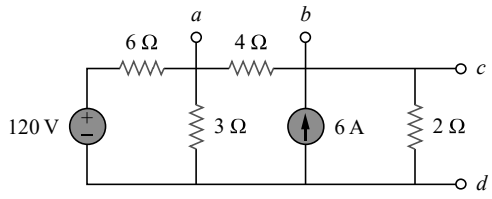


Figure 4.101 For Prob. 4.41.

*An asterisk indicates a challenging problem.

4.42 For the transistor model in Fig. 4.102, obtain the Thevenin equivalent at terminals *a-b*.

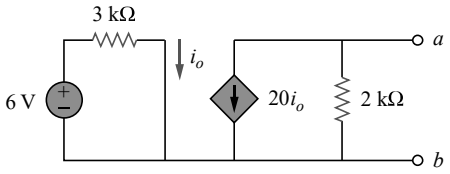


Figure 4.102 For Prob. 4.42.

4.43 Find the Norton equivalent at terminals *a-b* of the circuit in Fig. 4.103.

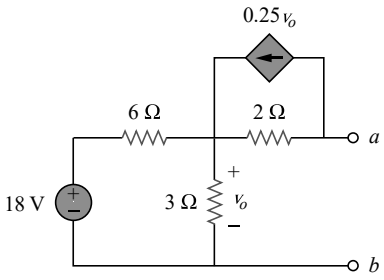


Figure 4.103 For Prob. 4.43.

*__4.44__ Obtain the Norton equivalent at terminals *a-b* of the circuit in Fig. 4.104.

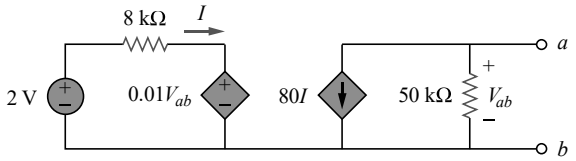


Figure 4.104 For Prob. 4.44.

4.45 Use Norton's theorem to find current i in the circuit of Fig. 4.80.

4.46 Obtain the Thevenin and Norton equivalent circuits at the terminals *a-b* for the circuit in Fig. 4.105.

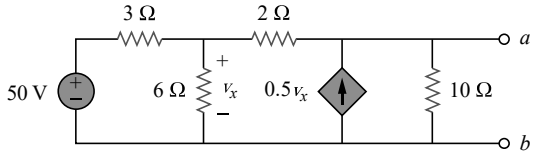


Figure 4.105 For Probs. 4.46 and 4.65.

4.47 The network in Fig. 4.106 models a bipolar transistor common-emitter amplifier connected to a load. Find the Thevenin resistance seen by the load.

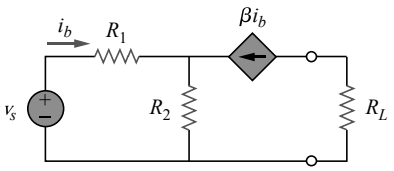


Figure 4.106 For Prob. 4.47.

4.48 Determine the Thevenin and Norton equivalents at terminals *a-b* of the circuit in Fig. 4.107.

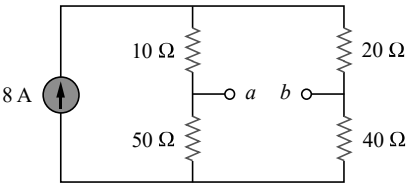


Figure 4.107 For Probs. 4.48 and 4.66.

*__4.49__ For the circuit in Fig. 4.108, find the Thevenin and Norton equivalent circuits at terminals *a-b*.

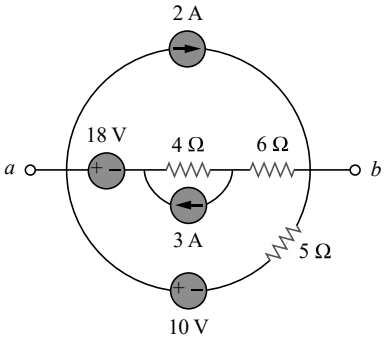


Figure 4.108 For Probs. 4.49 and 4.67.

*__4.50__ Obtain the Thevenin and Norton equivalent circuits at terminals *a-b* of the circuit in Fig. 4.109.

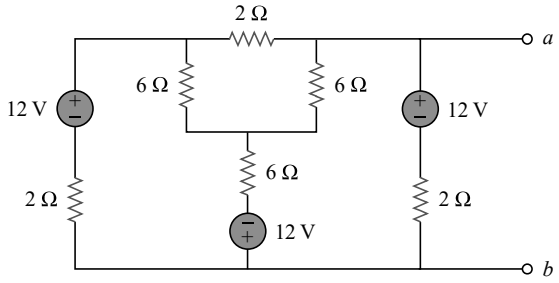


Figure 4.109 For Prob. 4.50.

*4.51 Find the Thevenin equivalent of the circuit in Fig. 4.110.

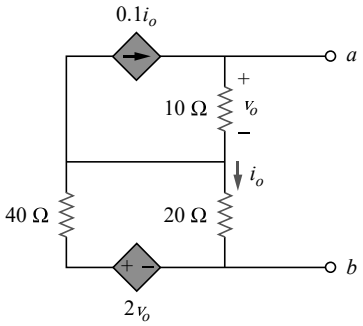


Figure 4.110 For Prob. 4.51.

4.52 Find the Norton equivalent for the circuit in Fig. 4.111.

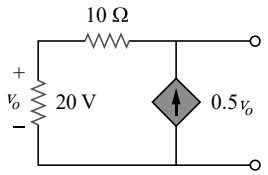


Figure 4.111 For Prob. 4.52.

4.53 Obtain the Thevenin equivalent seen at terminals *a-b* of the circuit in Fig. 4.112.

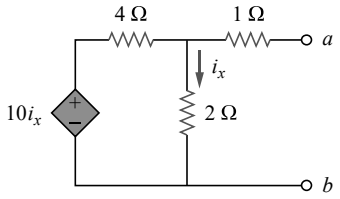


Figure 4.112 For Prob. 4.53.

Section 4.8 Maximum Power Transfer

4.54 Find the maximum power that can be delivered to the resistor R in the circuit in Fig. 4.113.

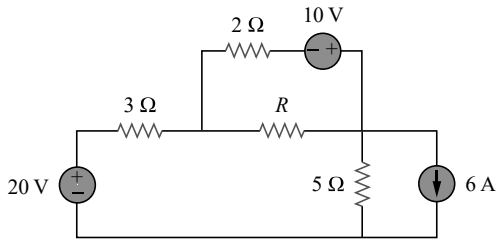


Figure 4.113 For Prob. 4.54.

4.55 Refer to Fig. 4.114. For what value of R is the power dissipated in R maximum? Calculate that power.

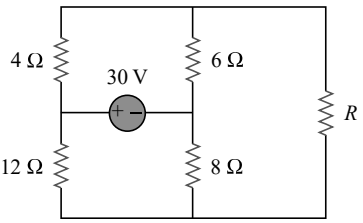


Figure 4.114 For Prob. 4.55.

*4.56 Compute the value of R that results in maximum power transfer to the 10-Ω resistor in Fig. 4.115. Find the maximum power.

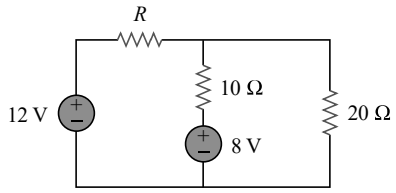


Figure 4.115 For Prob. 4.56.

4.57 Find the maximum power transferred to resistor R in the circuit of Fig. 4.116.

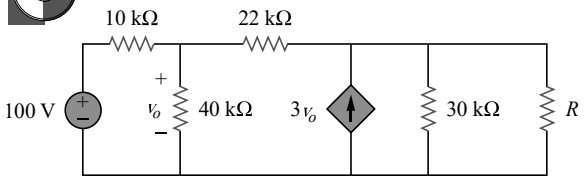


Figure 4.116 For Prob. 4.57.